A THEOREM OF LUSIN

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Part I

1. Let

$$f(z) = \sum_{n=0}^{\infty} a_n z^n \tag{1}$$

be a function holomorphic in the circle $|z| < 1$. The function $f(z)$ is said to belong to the class H^{λ}, $\lambda > 0$, if the expression

$$I_{\lambda}(r) = I_{\lambda}(r, f) = \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^{\lambda} d\theta \tag{2}$$

is bounded for $r < 1$. It is well known[1] that, if $f(z)$ belongs to H^{λ}, then for almost every θ the limit

$$f(e^{i\theta}) = \lim_{z \to e^{i\theta}} f(z) \tag{3}$$

exists, where z tends to $e^{i\theta}$ along any non-tangential path. Hence, if C denotes the upper bound of the expression (2) for $0 \leqq r < 1$, we have

$$\int_0^{2\pi} |f(e^{i\theta})|^{\lambda} d\theta \leqq C. \tag{4}$$

In the sequel we shall also use the fact that the expression $I_{\lambda}(r)$ is a non-decreasing function of r, and so in particular

$$I_{\lambda}(r) \leqq \frac{1}{2\pi} \int_0^{2\pi} |f(e^{i\theta})|^{\lambda} d\theta \qquad (0 \leqq r < 1). \tag{5}$$

If $\lambda \geqq 1$, the real part and the imaginary part of the power series (1) on the circle $|z| = 1$ are both Fourier series of functions of the class L^{λ}.

Let Ω denote the interior of a simple closed curve Γ given by the equation

$$\rho = \psi(\theta) \qquad (-\pi \leqq \theta \leqq \pi) \tag{6}$$

and possessing, among others, the following two properties:

(i) Γ passes through the point $z = 1$, but otherwise lies entirely in the circle $|z| < 1$;

(ii) Γ is not tangent to the circle $|z| = 1$ at the point $z = 1$, that is, there

Received March 1, 1938.

[1] See F. Riesz, *Über die Randwerte einer analytischen Funktion*, Math. Zeitschr., vol. 18 (1922), pp. 87–95.