THE PROJECTIONS OF THE ASYMPTOTIC CURVES

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1. **Introduction.** A line l_1 through a point of a surface in ordinary space but not lying in the tangent plane of the surface at the point and a line l_2 lying in the tangent plane but not passing through the point are called reciprocal lines if they are reciprocal polars with respect to the quadric of Lie at the point.

G. M. Green,[1] in his investigation of the theory of reciprocal congruences, arrived at an important pair of reciprocal lines, now commonly called the canonical edges of Green, by considering the projections of the asymptotic curves upon the tangent plane at a point of a surface.

In this paper we propose to continue the investigation of the projections of the asymptotic curves upon the tangent plane at a point of a surface. For this purpose power series expansions for the projected asymptotics are deduced, the center of projection being a point on an arbitrary line l_1 at a point of the surface. Consideration of certain osculants associated with the projected asymptotic curves leads to new geometric characterizations of the canonical edges of Green and to other canonical lines. Finally, brief attention is given to a particular transformation of Čech.

2. **The projections of the asymptotic curves.** If the four homogeneous projective coördinates $x^{(1)}, \cdots, x^{(4)}$ of a point P_x on a non-ruled surface S in ordinary space are given as analytic functions of two independent variables u, v, and if the parametric net on S is the asymptotic net, then the functions x are solutions of a system of differential equations which, by suitable choice of proportionality factor, can be reduced to Fubini's canonical form

$$\begin{aligned} x_{uu} &= px + \theta_u x_u + \beta x_v, \\ x_{vv} &= qx + \gamma x_u + \theta_v x_v, \end{aligned} \qquad (\theta = \log \beta\gamma). \tag{1}$$

The coefficients of these equations are functions of u, v and satisfy three integrability conditions.

The coördinates X of a point near P_x and on the u-curve through P_x are given by an expansion of the form

$$X = x + x_u \Delta u + \tfrac{1}{2} x_{uu} \Delta u^2 + \cdots. \tag{2}$$

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[1] G. M. Green, *Memoir on the general theory of surfaces and rectilinear congruences*, Transactions of the American Mathematical Society, vol. 20(1919), p. 108.