A GENERALIZATION OF MULTIPLE SEQUENCE TRANSFORMATIONS

By Hugh J. Hamilton

1. **Introduction and statement of purpose.** It is the purpose of the present paper[1] to extend the definitions of the classes of complex multiple sequences $\{s_{k^{(1)},k^{(2)},\ldots,k^{(n)}}\}$ considered in H_1 so that they may have meaning for complex functionals f of l variables $x^{(1)}, x^{(2)}, \cdots, x^{(l)}$, where, for $\nu = 1, 2, \cdots, l$, $x^{(\nu)}$ is the general element of an aggregate $\mathfrak{E}^{(\nu)}$ of considerable generality, interpretable, in particular, either as the positive integers or as the points of a Euclidean space; and to derive, by means of the results of $H_{1,2}$, conditions on the n-dimensional matrix $||f_{k^{(1)},k^{(2)},\ldots,k^{(n)}}||$ of complex functionals of the l variables $x^{(1)}, x^{(2)}, \cdots, x^{(l)}$, necessary and sufficient for the various sequence class-to-functional class transformations under the relation

$$F(x^{(1)}, x^{(2)}, \cdots, x^{(l)}) \equiv \sum_{k^{(1)},k^{(2)},\cdots,k^{(n)}=1}^{\infty} f_{k^{(1)},k^{(2)},\ldots,k^{(n)}}(x^{(1)}, x^{(2)}, \cdots, x^{(l)}) s_{k^{(1)},k^{(2)},\ldots,k^{(n)}}$$

analogous to the sequence class-to-sequence class transformations considered in $H_{1,2}$.

In order to clarify ideas, consider the solution of the problem: to find conditions on the linear matrix of complex functionals $||f_i||$ necessary and sufficient that $F(t) \equiv \sum_{i=1}^{\infty} f_i(t)s_i$ exist finite for each t in $(0, \infty)$ and converge to 0 as t tends to infinity, whenever $\{s_i\}$ is a null sequence of complex numbers.

Denote by S the class of non-negative, real sequences $\{t_j\}$ for which $\lim_j t_j = \infty$. Now $\lim_{t\to\infty} F(t) = 0$ if and only if $\lim_j F(t_j) = 0$ for each $\{t_j\} \epsilon S$. Hence the desired conditions are simply that

$$(\alpha) \qquad \sum_{i=1}^{\infty} |f_i(t_j)| < B(\{t_j\}) \text{ for each } j \qquad (\text{each } \{t_j\} \epsilon S);$$

$$(\beta) \qquad \lim_j f_i(t_j) = 0 \text{ for each } i \qquad (\text{each } \{t_j\} \epsilon S),$$

as follows from well-known theorems of the Silverman-Toeplitz type (or from $H_{1,2}$); or, more elegantly, that

Received November 29, 1937; presented to the American Mathematical Society, April 16, 1938.

[1] This paper assumes familiarity with the contents of the preceding note, *Change of dimension in sequence transformations*; hence also with the paper therein cited as H_1. The paper H_1, as revised in the note, will be referred to as $H_{1,2}$.