THEOREMS ON FOURIER SERIES AND POWER SERIES

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1. Introduction

1.1. **Notation.** If $p > 0$, we write

$$\mathfrak{S}_p[a_n] = \left(\sum_{-\infty}^{\infty} | a_n |^p\right)^{1/p},$$

$$\mathfrak{T}_p[c_n] = \left(\int_0^1 \left(\sum_0^{\infty} | c_n | x^n\right)^p dx\right)^{1/p},$$

$$\mathfrak{M}_p[F(\theta)] = \left(\int_{-\pi}^{\pi} | F(\theta) |^p d\theta\right)^{1/p}.$$

If $g(z)$ is a regular analytic function for $| z | < 1$, we write

$$\mathfrak{H}_p[g(z)] = \lim_{r \to 1} \left(\int_{-\pi}^{\pi} | g(re^{i\theta}) |^p d\theta\right)^{1/p}.$$

(The limit exists, since the expression in the bracket increases with r.)

1.2. Suppose that $F(\theta)$ is periodic and integrable and that $g(z)$ is regular in $| z | < 1$. Let

$$F(\theta) \sim \sum_{-\infty}^{\infty} a_n e^{ni\theta} \qquad (a_0 = 0), \tag{1.2.1}$$

$$g(z) = \sum_1^{\infty} c_n z^n \qquad (| z | < 1). \tag{1.2.2}$$

We shall prove that if p, q, α satisfy certain conditions,

$$\mathfrak{S}_q[c_n n^{-\lambda}] \leqq K \mathfrak{H}_p[g(z)(1 - z)^\alpha], \tag{1.2.3}$$

$$\mathfrak{S}_q[a_n n^{-\lambda}] \leqq K \mathfrak{M}_p[F(\theta)\theta^\alpha], \tag{1.2.4}$$

where

$$\lambda = \frac{1}{p} + \frac{1}{q} + \alpha - 1, \tag{1.2.5}$$

and the constants $K(p, q, \alpha)$ are independent of $g(z)$ and $F(\theta)$.

Special cases of these inequalities, due to Hausdorff[1] and Hardy and Little-

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[1] Hausdorff [5], Theorem II. (Numbers in brackets refer to the references at the end of the paper.) This is the case $\alpha = \gamma = 0$ of (1.2.4).