TAUBERIAN THEOREMS RELATED TO BOREL AND ABEL SUMMABILITY

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The following work is divided into two sections. The first deals with a Tauberian theorem of summability methods related to Borel summability, and the second with a Tauberian theorem of summability methods related to Abel summability.

In 1925 Robert Schmidt gave a proof of the following theorem [**6**]:[1]

If a series is Abel summable to the value s, and if the partial sums s_n satisfy the condition

$$\underline{\lim}\,(s_m - s_n) \geqq 0 \quad \textit{whenever} \quad \frac{m-n}{n} \to 0 \qquad (m > n),$$

then

$$\lim_{n\to\infty} s_n = s.$$

In the same year Schmidt gave a proof of an analogous theorem concerning Borel summability. This theorem states [**7**]:

If a series is Borel summable to the value s, and if the partial sums s_n satisfy the condition

$$\underline{\lim}\,(s_m - s_n) \geqq 0 \quad \textit{whenever} \quad \frac{m-n}{n^{\frac{1}{2}}} \to 0 \qquad (m > n),$$

then

$$\lim s_n = s.$$

In the following two years Vijayaraghavan [**10**, **11**] gave a new and more elementary proof for each of these theorems.

It will be observed that in each of the preceding cases the method of summability is a power series method, and that the condition imposed on the sequence of partial sums is of the following type:

$$\text{(i)} \qquad \underline{\lim}\,(s_m - s_n) \geqq 0 \quad \text{whenever} \quad \frac{m-n}{\varphi(n)} \to 0 \qquad (m > n),$$

where $\varphi(n)$ is an increasing function of n which tends to ∞ with n.

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[1] The numbers refer to the list of references at the end of this paper. For an extensive bibliography on the subject of Tauberian theorems see N. Wiener, *Tauberian theorems*, Annals of Mathematics, (2), vol. 33 (1932), pp. 1-100.