ON BERNOULLI'S NUMBERS AND FERMAT'S LAST THEOREM

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1. **Introduction.** In the present article a report will be given on the work which has been carried out under a grant made to the writer from the Penrose Fund of the American Philosophical Society.

One of the objects of the work was to extend the known tables of Bernoulli numbers expressed as rational fractions in their lowest terms and to check all previous tables. This was carried out by D. H. Lehmer,[1] who tabulated B_n for $n = 91$ to 110, inclusive. The previous tables had given the values B_n $(n = 1, 2, \cdots, 92)$. The first ninety of these were reproduced in the tables of H. T. Davis.[2] Here

$$B_a = (-1)^{a-1} b_{2a}$$

and

$$(b + 1)^n = b_n \qquad (n \neq 1),$$

where the expression on the left means that $(b + 1)$ is taken to the n-th power by means of the binomial theorem, and b_k is substituted for b^k $(k = 1, 2, \cdots, n)$. All the above mentioned B's were employed by Lehmer in checking the regularity of primes, a prime p being defined as regular if it does not divide the numerators of any of the first $(p - 3)/2$ B's. The details of these latter computations will be treated below.

Another object of the work under the grant was to extend the results of the writer on Fermat's Last Theorem for special exponents.

In other papers[3] it was established that

$$x^l + y^l + z^l = 0 \tag{I}$$

is impossible for x, y, and z non-zero rational integers and l a given integer $2 < l < 307$. In the present article we shall describe the work which established this result for $306 < l < 617$, excepting $l = 587$ which exponent has not yet been tested (cf. note, p. 584). As the methods employed for these large exponents are quite elaborate and complicated, we shall explain many details.

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[1] This Journal, vol. 2 (1936), pp. 460–464. This article includes references to previous tables. Lehmer computed the value of B_{196} in addition to those mentioned but not as part of the present project. Cf. Annals of Math., vol. 36 (1935), p. 648.

[2] *Tables of Higher Mathematical Functions*, vol. 2, Bloomington, Indiana, 1935, pp. 230–233.

[3] Proc. Natl. Acad. Sci., vol. 17 (1931), pp. 661–673 (referred to later as N. A.) with references there given.