TRANSFORMATION OF DIFFERENTIAL EQUATIONS IN THE NEIGHBORHOOD OF SINGULAR POINTS

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1. Introduction. The singular points of the system of differential equations

$$\frac{dx}{dt} = X(x, y), \qquad \frac{dy}{dt} = Y(x, y), \tag{1}$$

where the functions $X(x, y)$ and $Y(x, y)$ both vanish at a point, have been considered by many mathematicians. Beginning with Briot and Bouquet the list includes Poincaré, Picard, Bendixson, and continues with Dulac, Malmquist, Perron, Birkhoff, and many others. A full bibliography, particularly for complex differential equations, can be found in Mémorial des Sciences Mathématiques, Fascicule LXI, *Points singuliers des équations différentielles*, by H. M. Dulac.

The discussion here assumes the functions X and Y to be real and analytic in x and y, in the neighborhood of the singular point. The variables x, y, t take on only real values, and only real transformations are introduced. Furthermore, it is assumed that the singular point considered is at the origin and that the first degree terms in $X(x, y)$ and $Y(x, y)$ are such that the differential equation can be transformed by a linear transformation to the form[1]

$$\begin{aligned} \frac{dx}{dt} &= -y + \sum_{i,j=0}^{\infty} a_{ij}x^i y^j = X(x, y), \\ \frac{dy}{dt} &= x + \sum_{i,j=0}^{\infty} b_{ij}x^i y^j = Y(x, y) \qquad (i + j \geqq 2). \end{aligned} \tag{1.1}$$

At such a point the solutions are closed curves or spirals and the point is called a center or a focal point, respectively.

It is proposed here to find a canonical form for the system (1.1) and to discuss the properties of the transformation attaining that form.

2. Failure of the usual method. It appears from the discussion below that it is not always possible to set up formal power series

$$\begin{aligned} u &= x + \sum c_{ij}x^i y^j = f(x, y), \\ v &= y + \sum d_{ij}x^i y^j = g(x, y) \end{aligned} \tag{2}$$

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[1] Poincaré, Jour. de Math., (4), vol. 1, p. 172; Picard, *Traité d'Analyse*, 1928, Chap. IX.