# POLAR CORRESPONDENCE WITH RESPECT TO A CONVEX REGION

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We denote as polar correspondence (abbreviated P.C.) with respect to a convex region $R$ in projective $n$-dimensional space $\pi_n$ any one-to-one correspondence of the points of $R$ and the hyperplanes outside $R$. The study o such a correspondence is essentially the study of a contragredient vector with special consideration of the convexity of the domain of definition. In §1 of this paper the representation of a P.C. in homogeneous coördinates is discussed. A P.C. is called *positive* if a point and its polar plane are not separated by any other point and its polar plane. In §2 it is proved that every positive P.C. is continuous. §§3–4 deal with *symmetric* P.C.'s; a P.C. is called symmetric if in the neighbourhood of every point $P$ it is approximated by an ordinary polar correspondence with respect to a quadric, denoted as the *tangential* quadric in $P$. A positive symmetric P.C. may be generated by a convex hypersurface in $(n + 1)$-dimensional space $\pi_{n+1}$ in such a way that the line joining any point $Q$ of the surface to a fixed point of $\pi_{n+1}$ and the tangential plane in $Q$ intersect $\pi_n$ in a point and its polar respectively.

In the remainder of the paper a general class of P.C.'s is discussed, which are generated by continuous positive mass distributions on $R$. Given any hyperplane $p$ outside $R$, the pole of $p$ shall be that point which becomes the center of mass of $R$ with the given mass distribution, in case $p$ is chosen as plane at infinity. In §4 it is proved that a P.C. generated in this way is always positive and symmetric. In §§5–6 it is shown that the tangential quadric at a point $P$ is identical with Legendre's ellipsoid of inertia of $R$ if the polar of $P$ is plane at infinity. Moreover, some inequalities involving $R$ and its tangential quadrics are given.[1]

1. Let $R$ be an *open* convex region in projective $n$-dimensional space $\pi_n$; i.e., an open set with the following properties:

(1) If $P$ and $Q$ are points of $R$, one of the two straight line segments bounded by $P$ and $Q$ belongs to $R$;

(2) If $S$ denotes the set of points which are neither points of $R$ nor boundary points of $R$, there is at least one hyperplane in $S$.

Definition. A one-to-one correspondence between the points of $R$ and

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[1] I am indebted to the referee for pointing out that the methods used in §1 are closely related to those used by Steinitz in his paper *Bedingt konvergente Reihen und konvexe Systeme* in Crelle's Journal; cf. in particular vol. 146, p. 32 et seq., where Steinitz deals with convex regions in projective space. Our sets $\rho$ and $\bar{\rho}$ appear there as number sets $A$ and $-A$, and theorems corresponding to our Theorems 1.7 and 1.8 are given.