A CLASS OF LINEAR GROUPS WITH INTEGRAL COEFFICIENTS

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Introduction. In the study of certain groups of Cremona transformations in S_k which I have called "regular groups" ([1]II, §§4, 5), the determination of the types of transformations in the Cremona group was accomplished by the use of a related linear group with integral coefficients. In later papers[2] it appeared that the class of Cremona groups with such related linear groups is probably quite extensive. It is the purpose of the present paper to discuss linear groups of this character independently of their association with Cremona groups. They are generated by a finite number of involutorial elements of a particular type, derived in §1, and characterized more completely in §2. These groups divide into two classes according to the values of a certain constant, $e = \pm 1$, and they have an additional integer parameter ϵ. The linear groups first mentioned occur when $\epsilon = 1$, $e = -1$. These occurred in pairs of "associated groups" and in §3 this isomorphism described as association is extended to generic ϵ.

Each group has an invariant linear, and an invariant quadratic, form. This imposes the necessary conditions on the coefficients of the generic element, which are obtained in §4. These conditions, however, are not sufficient. The de Jonquières subgroups, defined in §5 after the manner of the de Jonquières group of planar Cremona transformations, are used in §§7, 8 to separate the cases of finite and of infinite order.

The types of symmetric transformations, already determined when $\epsilon = 1$, $e = -1$, are found for the general case in §6.

A particular class, $g_\rho(\alpha)$, of these groups, whose generators are unusually simple, is studied in §9 with particular reference to aggregates of products of ternary de Jonquières transformations, and again in §10 with reference to the nature of the coefficients of its elements. The writer hopes to consider the more general group in a later paper.

1. **A particular type of involutorial matrix.** In connection with the so-called "symmetric Cremona transformations" there occur linear transformations with integer coefflcients, the matrix of the coefficients having the particular form

$$
(1) \qquad \begin{array}{cccc}
\alpha & -\beta & -\beta & -\beta \cdots \\
\delta & -\gamma & -\epsilon & -\epsilon \cdots \\
\delta & -\epsilon & -\gamma & -\epsilon \cdots \\
\delta & -\epsilon & -\epsilon & -\gamma \cdots \\
\cdots & \cdots & \cdots & \cdots
\end{array}
$$

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[1] A. B. Coble, *Point sets and Cremona groups*, I: Trans. Amer. Math. Soc., vol. 16 (1915), pp. 155–198; II: Trans. Amer. Math. Soc., vol. 17 (1916), pp. 345–385.

[2] A. B. Coble, *Groups of Cremona transformations in space of planar type*, I: this Journal, vol. 2 (1936), pp. 1–9; II: this Journal, vol. 2 (1936), pp. 205–219.