ABELIAN GROUPS WITHOUT ELEMENTS OF FINITE ORDER*

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An abelian group which is written so that its symbols are combined by addition and which has no elements of finite order other than 0[1] may be called completely reducible, if it is a direct sum of groups of rank one. For every group is contained in a completely reducible group of the same rank. There exist furthermore direct irreducible groups of every finite rank and the groups of rank 1 are exactly the subgroups of the additive group of the rational numbers and therefore irreducible.

The structure of a completely reducible group is uniquely determined by the ranks of the differences of certain characteristic subgroups. A survey of the structures of all subgroups of completely reducible groups would involve the solution of the general structure problem, since every group is contained in a completely reducible group. But it is possible to characterize a class of completely reducible subgroups (of completely reducible groups) which are isomorphic with a direct summand of the whole group.

Every property of a completely reducible group which refers to finite subsets or to subgroups of finite rank also holds true for separable groups, i.e., for groups whose finite subsets are contained in completely reducible direct summands. Countable separable groups are completely reducible. But there exist separable groups which are not completely reducible, e.g., vector groups like the additive group of all the sequences of integers. Further criteria for complete reducibility, for separability and for the complete reducibility of separable groups are given. There exists in particular a characterization of the direct summands of finite rank which holds true in every separable group and which is valid in a group of finite rank if, and only if, this group is completely reducible. Furthermore, every direct summand of finite rank of a separable group is completely reducible.

The subsets b' and b'' of the group J are isotype in the group J, if there exists a proper automorphism of J which maps b' upon b''. The classes of isotype elements of a separable group are determined by complete sets of invariants and an enumeration of the characteristic and of the strictly characteristic subgroups is based on this classification of the elements. The existence of characteristic subgroups which are not strictly characteristic and the existence of elements which are not isotype though contained in the same characteristic and in the same strictly characteristic subgroups are closely related phenomena; but neither is a consequence of the other.

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[1] The word "group" is substituted for this longer statement wherever there is no danger of confusion.