

# STRESSES IN MODERATELY THICK RECTANGULAR PLATES

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## 1. Introduction

In 1922 G. D. Birkhoff<sup>1</sup> suggested a method for solving plate problems which involves the representation of the displacements by power series. C. A. Garabedian<sup>2</sup> and H. W. Sibert<sup>3</sup> used this idea in developing methods for solving problems in moderately thick circular plates. Garabedian<sup>4</sup> has also published some results for uniformly loaded rectangular plates.

The authors give a solution for the displacements in an elastic isotropic moderately thick rectangular plate under the action of any given load which can be expressed as a polynomial in  $x, y$  continuous over the entire plate and with prescribed boundary conditions at the edges. The method, similar to that used by Sibert<sup>5</sup> for circular plates, is based on the assumption that the components of displacement can be developed in positive integral powers of  $z$ . In this type of problem, the displacements must satisfy (a) the stress equations of equilibrium throughout the plate, (b) the surface traction conditions on the upper and lower faces, (c) the boundary conditions at the edges.

## 2. General theory

a. **Form of the displacements.** The displacements,  $u, v, w$ , are given by

$$(1) \quad u = \sum_{n=0}^{\infty} U_n \frac{z^n}{n!}, \quad v = \sum_{n=0}^{\infty} V_n \frac{z^n}{n!}, \quad w = \sum_{n=0}^{\infty} W_n \frac{z^n}{n!},$$

where  $U_n, V_n$ , and  $W_n$  are continuous and continuously differentiable functions of  $x, y$ . The equations of equilibrium are (A. E. H. Love,<sup>6</sup> p. 134)

$$(2) \quad (\lambda + \mu) \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \Delta + \mu \nabla^2 (u, v, w) = 0.$$

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<sup>1</sup> G. D. Birkhoff, *Circular plates of variable thickness*, Phil. Mag., vol. 43 (1922), pp. 953-962.

<sup>2</sup> C. A. Garabedian, *Circular plates of constant or variable thickness*, Trans. Amer. Math. Soc., vol. 25 (1923), pp. 343-398.

<sup>3</sup> H. W. Sibert, *Moderately thick circular plates with plane faces*, Trans. Amer. Math. Soc., vol. 33 (1931), pp. 329-369.

<sup>4</sup> C. A. Garabedian, *Comptes Rendus, Paris*, vols. 178 (1924), 180 (1925), 181 (1925), 186 (1928), 195 (1932).

<sup>5</sup> Loc. cit.

<sup>6</sup> In this paper all references to Love are to the fourth edition of his *Mathematical Theory of Elasticity*, 1927.