ON THE FOURIER TRANSFORMS OF DISTRIBUTIONS ON CONVEX CURVES

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In a previous paper,[1] the asymptotic formula for the Bessel function J_0 has been applied to the derivation of smoothness properties of infinite convolutions of circular equidistributions. For this purpose, not the asymptotic formula but merely an appraisal was needed. It has been indicated in that paper[2] that the same method is valid also in the case of infinite convolutions of certain distribution functions along convex curves—in particular, in the case of some asymptotic distribution problems connected with the Riemann zeta function. The necessary appraisal of the function corresponding in this more general case to the function J_0 has then been carried through[3] by a simple application of a lemma of van der Corput and Landau.[4] The object of the present paper is to replace this appraisal by an asymptotic formula. While the former corresponds to $J_0(r) = O(r^{-\frac{1}{2}})$, $r \to \infty$, the latter will be a generalization of

$$J_0(r) = r^{-\frac{1}{2}}(2/\pi)^{\frac{1}{2}} \cos (r - \pi/4) + O(r^{-1}), \qquad r \to \infty.$$

The general function to be considered is, in contrast to the particular function $J_0(r)$, a function not only of r but of an angular parameter ψ also. For a fixed value of ψ, the asymptotic formula in question may be obtained by applying an elementary method.[5] What is needed for the applications mentioned above, and what will be proved in what follows, is the fact that the asymptotic formula holds uniformly for all values of ψ, i.e., that the error term is in absolute value less than Cr^{-1}, where C is a constant independent both of r and of ψ.

Let $x = x(\varphi)$, $y = y(\varphi)$, where $0 \leqq \varphi < 2\pi$, be a parametric representation of a convex Jordan curve S in the (x, y)-plane. It will be described more precisely below. Let $\sigma = \sigma(E)$ be an absolutely additive set function defined, for every Borel set E of the (x, y)-plane, by setting $\sigma(F)$ equal to $1/(2\pi)$ times the linear measure of those φ for which $(x(\varphi), y(\varphi))$ is contained in FS, if F is any open set in plane. In particular, it is seen that S is the spectrum[6] of σ. By

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[1] A. Wintner, loc. cit., I. The references are collected at the end of the paper.

[2] A. Wintner, ibid., pp. 328–329.

[3] B. Jessen and A. Wintner, loc. cit., Theorem 12, p. 63.

[4] Cf., e.g., R. Kershner, loc. cit., where further references are given.

[5] Cf. A. Wintner, loc. cit., III, pp. 57–60, where references to the literature are given.

[6] For the definition of the spectrum, together with some properties of spectra, cf. A. Wintner, loc. cit., II, pp. 9–10, and E. K. Haviland, loc. cit., II, pp. 653–654. It has been pointed out by Professor Khintchine that, contrary to statements in these papers, the vectorial sum of two closed sets is necessarily closed only when at least one of these sets is