ABEL-POISSON SUMMABILITY OF DERIVED CONJUGATE FOURIER SERIES

By A. F. Moursund

1. **Introduction.**[1] In this paper we give theorems concerning the Abel-Poisson summability of the r-th, $r = 0, 1, 2, \cdots$, derived series of the conjugate series of the Fourier series.[2] These theorems may be considered as extensions of theorems given by B. N. Prasad,[3] A. Plessner,[4] and others for the summability of the conjugate series and its first derived series.

2. **Notation.** Throughout this note we assume that the function $f(x)$ is Lebesgue integrable on $(-\pi, \pi)$ and of period 2π. The letters r and p always represent positive integers or zero, and the letter K represents a positive absolute constant which need not be always the same even in a single discussion. For convenience we designate a fixed value of x by x instead of the usual x_0.

We set

$$p(v, s) \equiv (1 + s^2 - 2s \cos v)^{-1}, \qquad P(v, s) \equiv sp(v, s) \sin v; \tag{1}$$

$$M_r^{(p)}(v, s) \equiv v^{r+p+1} \frac{\partial^{r+p+1}}{\partial v^{r+p+1}} \{P(v, s) - 1/2 \cot v/2\}; \tag{2}$$

$$V(s, x) \equiv \sum_{n=1}^{\infty} (-b_n \cos nx + a_n \sin nx)s^n \qquad (0 \leqq s < 1), \tag{3}$$

Received September 5, 1935; presented to the American Mathematical Society, September 13, 1935.

[1] The author is indebted to the referee for many helpful suggestions.

[2] No theorems concerning the Abel-Poisson summability of the r-th, $r > 1$, derived conjugate series appear in the literature. For theorems concerning the summability of such series by other methods see the following papers: A. F. Moursund, *On summation of derived series of the conjugate Fourier series*, Annals of Mathematics, (2), vol. 36 (1935), pp. 182–193, and American Journal of Mathematics, vol. 57 (1935), pp. 854–860; A. H. Smith, *On the summability of derived conjugate series of the Fourier-Lebesgue type*, Bulletin of the American Mathematical Society, vol. 40 (1934), pp. 406–412; A. F. Moursund, *On the r-th derived conjugate function*, Bulletin of the American Mathematical Society, vol. 41 (1935), pp. 131–136. Since Abel-Poisson summability follows from Cesàro summability it is possible to obtain theorems which are similar to, but less general than, some of the theorems of this note from Theorem 6.4 of the second paper cited here. Theorem 1 does not follow from any of the results of papers listed here.

[3] B. N. Prasad, *Contribution à l'étude de la série conjuguée d'une série de Fourier*, Journal de Mathématiques Pures et Appliquées, (9), vol. 11 (1932), pp. 153–205. This paper gives a long bibliography.

[4] A. Plessner, *Zur Theorie der konjugierten trigonometrischen Reihen*, Mitteilungen des Mathematischen Seminars der Universität Giessen, Heft 10 (1923).