MOMENTS OF INERTIA OF CONVEX REGIONS

By Fritz John

Let R denote a closed and bounded two-dimensional convex region. Let d be the greatest, Δ the smallest diameter of R, a diameter being defined as the distance of two parallel lines of support.[1] Let A be the area and L the circumference of R. It was recently proved by F. Behrend that there exist for any R affine transformations transforming R into convex regions for which any one of the following inequalities is satisfied:

$$\frac{d}{\Delta} \leqq \sqrt{2}, \qquad \frac{A}{\Delta^2} \leqq 1, \qquad \frac{d}{L} \leqq \tfrac{1}{4}\sqrt{2};$$

if, moreover, R has a center, i.e., if R is symmetrical with respect to some point, then there are also affine transformations transforming R into regions for which any of the following inequalities hold:

$$\frac{d^2}{A} \leqq 2, \qquad \frac{L^2}{A} \leqq 16, \qquad \frac{L}{\Delta} \leqq 4.$$

The corresponding equalities are all satisfied in the case of a square.

Now let λ denote the ratio of the major and minor axes of the ellipse of inertia of R corresponding to the center of mass of R in a homogeneous mass distribution, i.e., of the "central" ellipse of inertia of R. We shall prove in this paper that the inequalities

$$(1) \qquad \frac{d}{\Delta\lambda} \leqq \sqrt{2} \qquad\qquad (2) \qquad \frac{A}{\Delta^2\lambda} \leqq 1$$

hold; if R has a center, then also

$$(3) \qquad \frac{d^2}{A\lambda} \leqq 2.$$

These inequalities include some of Behrend's results; for every R can be easily transformed by an affine transformation into a region for which the central ellipse of inertia is a circle, i.e., for which $\lambda = 1$, and in this case $d/\Delta \leqq \sqrt{2}$, $A/\Delta^2 \leqq 1$, and if R has a center $d^2/A \leqq 2$ also.

In a second paper I intend to show (1) that if R has a center,

$$\frac{d}{\Delta\lambda} > \frac{\sqrt{2 + \sqrt[3]{100}}}{3};$$

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[1] For notations see *Theorie der konvexen Körper* by Bonnesen and Fenchel; we shall refer to this book as B.-F.