APPLICABILITY WITH PRESERVATION OF BOTH CURVATURES

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1. **Introduction.** The determination of conditions necessary and sufficient that there exist a surface applicable to a given surface with preservation of both the total and mean curvatures constitutes a problem of classical differential geometry which has received no little attention.[1] In this paper, various new conditions, all in invariantive form, are found. The map of a surface satisfying these conditions on a surface applicable to it in the manner described is studied in some detail and is shown to have many interesting geometrical properties.

The treatment is by means of the invariant methods recently exploited by the author.[2] These methods are particularly advantageous in the present problem, in that they naturally disclose facts which otherwise might remain undiscovered or prove complicated to establish.

2. **Necessary and sufficient conditions.** Let there be given a surface[3] $S: x_i = x_i(u, v), i = 1, 2, 3$, and assume that there exists a surface $\bar{S}: \bar{x}_i = \bar{x}_i(u, v)$, $i = 1, 2, 3$, which is applicable to S so that both curvatures are preserved. Then any surface $\bar{S}^*$ which is symmetric to $\bar{S}$ is also applicable to S with preservation of both curvatures. Inasmuch as the sign of the mean curvature of a surface depends on the orientation of the directed normal, it follows that the normals to $\bar{S}$ and $\bar{S}^*$ must be so directed that corresponding directions of rotation about corresponding points have, with reference to these directed normals, opposite senses. Hence, for just one of the surfaces $\bar{S}$, $\bar{S}^*$, the map of the surface on S has the property that corresponding directions of rotation about corresponding points are the same. Without loss of generality we may assume that this is the surface $\bar{S}$; the surface $\bar{S}^*$ we then exclude completely, since it is readily obtainable from $\bar{S}$. In other words, we assume that the normals of two surfaces which are applicable to one another with preservation of both curvatures are so directed that (without invalidating the equality of the mean curvatures) corresponding directions of rotation about corresponding points, referred to these directed normals, have the same sense.

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[1] For references to the literature see Graustein, *Applicability with preservation of both curvatures*, Bull. Amer. Math. Soc., vol. 30 (1924), pp. 19–23. This paper will be referred to later as "Paper A".

[2] *Méthodes invariantes dans la géométrie infinitésimale des surfaces*, Mémoires de l'Académie Royale de Belgique (Classe des Sciences), (2), vol. 11 (1929); *Invariant methods in classical differential geometry*, Bull. Amer. Math. Soc., vol. 36 (1930), pp. 489–521. These papers will be referred to respectively as "B.M." and "I.M".

[3] It is assumed that all functions are real, single-valued, and analytic in a certain domain of the real variables u, v.