ON CERTAIN ANALYTIC CONTINUATIONS AND ANALYTIC HOMEOMORPHISMS

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1. Introduction. We generalize to the case of n complex variables and one real variable a theorem of Severi[1] regarding analytic continuation, over a limited domain[2] in the $(2n + 1)$-space of the variables, of a function given analytic near the boundary B. The theorem states that if B is connected the continuation is possible. Severi proves the theorem only for the case that $n = 1$ and the domain is of simple type. We remove all restrictions as to simplicity of the domain and its boundary.

The similar theorem for a region in the $2n$-space of $n > 1$ complex variables is Osgood's[3] extension of a theorem of Hartogs.[4] Because of certain geometric difficulties which seem not to be fully met in Osgood's proof, we give a detailed proof of this theorem. The proof applies without essential modification to the case of meromorphic continuation.[5]

As an application, we prove in the case of n complex variables that if the connected boundary of a limited domain in the space undergoes an analytic homeomorphism with non-vanishing jacobian, the transformation can be continued analytically over the domain to yield an analytic homeomorphism of the domain and its boundary (Theorem 4.II). A somewhat similar result is obtained for the case of one real and n complex variables (Theorem 4.III).

2. Functions of n complex variables. The following is the Osgood form of the theorem of Hartogs.

Theorem 2.I. *Let $\mathfrak{R}$ be a limited domain with connected boundary B in the $2n$-*

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[1] F. Severi, *Una proprietà fondamentale dei campi di olomorfismo di una variabile reale e di una variabile complessa*, Atti della Reale Accademia Nazionale dei Lincei, Rome, Rendiconti, (6), vol. 15 (1932), pp. 487–490. Our theorem is numbered 3.II.

[2] By a domain we mean an open set. A region is a connected open set. A limited point set is one of finite diameter.

[3] W. F. Osgood, *Lehrbuch der Funktionentheorie*, vol. 2, part I, Chapter 3, §11. We refer to the book as Osgood II.

[4] F. Hartogs, *Einige Folgerungen aus der Cauchyschen Integralformel bei Funktionen mehrerer Veränderlichen*, Sitzungsberichte der mathematisch-physikalischen Klasse der K. B. Akademie der Wissenschaften, München, vol. 36 (1906), pp. 223–241. Hartogs proves only that if a function is given defined over the entire region and boundary, analytic at the boundary and without removable singularities in the region, it is analytic in the region.

[5] Theorem 2.II. See Osgood II, Chapter 3, §13, and E. E. Levi, *Studii sui punti singolari essenziali delle funzioni analitiche di due o più variabili complesse*, Annali di Matematica, (3), vol. 17 (1910), pp. 61–87.