GROUPS OF CREMONA TRANSFORMATIONS IN SPACE OF PLANAR TYPE

BY ARTHUR B. COBLE

1. Introduction. We shall say that a group G of space Cremona transformations is *of planar type* if it possesses the distinguishing characteristics of the entire group of Cremona transformations in the plane. The characteristics which we shall stress are the following:

(α) G has an infinite continuous set of generators all of the same type.

In the plane this set is the set of quadratic transformations with distinct F-points.

(β) A particular element of G is defined by the choice of certain discontinuous parameters, positive or zero integers, which fix the *type* of the element (i.e., the nature of its F-system), and of certain continuous parameters which fix the position of its F-system.

This requirement rules out the group of inversions in space which has only two types of elements, namely: the collineation, and the quadratic transformation with a simple F-point and with a conic as an F-curve of the first kind.

(γ) Associated with G there is a group g of linear transformations on an unrestricted number of variables with integer coefficients. Each element of g defines a type of element in G. The product of two elements of G has a type defined by the product of the corresponding elements in g.

(δ) The linear group g of types in G has a linear and a quadratic invariant.

The number of groups of the type indicated which have thus far been exhibited is quite limited. In each space S_n ($n \geqq 2$) there is the group of "regular Cremona transformations",[1] which has interesting applications. These transformations have been called "punctual" by Miss Hudson.[2]

In S_3 there is a group whose generators are the cubic transformations which have a degenerate sextic F-curve of the first kind made up of a space cubic curve, fixed for the entire group, and of three variable bisecants of this curve. Montesano[3] has shown that in this group the types are isomorphic with the ternary types.

Snyder[4] reports a somewhat more special type of cubic transformation whose

Received December 20, 1935.

[1] A. B. Coble, *Point sets and allied Cremona groups* II, Transactions of the American Mathematical Society, vol. 17 (1916), pp. 345–385.

[2] Hilda P. Hudson, *Cremona Transformations in Plane and Space*, Cambridge University Press, 1927.

[3] D. Montesano, *Su alcuni tipi di corrispondenze cremoniane spaziali collegati alle corrispondenze birazionali piane di ordine n*, Napoli Rendiconti, (3), vol. 27 (1921), pp. 164–175.

[4] V. Snyder, *Some recent contributions to algebraic geometry*, Bulletin of the American Mathematical Society, vol. 40 (1934), pp. 673–687.