METABELIAN GROUPS AND TRILINEAR FORMS

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1. **The relation of the theory of metabelian groups to the fundamental problem of finite groups.** The theory of abstract groups is not yet one hundred years old. One hundred years is a short time in the history of mathematics, and yet the theory of groups is old compared with many subjects that are receiving the attention of large numbers of mathematicians. The greater part of the literature on finite groups is concerned with finding properties of certain known groups and with finding classes of groups which have certain given properties. This work, while necessary to the unfolding of the theory of groups, contributes usually only indirectly and often very remotely to the solution of the fundamental problem. The fundamental problem of finite groups is the determination of all the groups of a given order n. The determination of all the groups of a given order n will in general consist of the determination of several sets of properties such that each group of order n possesses all the properties of one set, and such that a group which possesses all the properties of one set does not possess all the properties of any other set. The determination of a definitive set of properties for a group does not close the subject of that particular group, for the reasons that there are in general large numbers of definitive sets for a given group and any given set may be entirely unsuitable for a particular purpose.

Although the fundamental problem was recognized and attacked at an early date, progress toward its solution has been very slow. In 1854 Cayley proved[1] that there are but two groups of order four and two groups of order six. In 1930 G. A. Miller listed the groups of orders up to 100 including those of order 64 which had not been completely determined before.[2] In 1934 Senior and Lunn[3] listed the groups from order 101 to 161 omitting those of order 128. Apparently nobody claims to have determined the groups of order p^7, and according to Miller's paper cited above Potron's attempt[4] to determine the groups of order p^6 was unsuccessful for $p = 2$. We do not wish to suggest that progress toward solution of the fundamental problem should be measured by that num-

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[1] *On the theory of groups as depending on the symbolical equation* $\theta^n = 1$, Philosophical Magazine, vol. 7 (1854), pp. 40–47; *Collected Mathematical Papers*, vol. II, pp. 123–130. For a sketch of the history of groups see the last paper in the forthcoming volume of the *Collected Works* of G. A. Miller, University of Illinois Press.

[2] *Determination of all the groups of order* 64, American Journal of Mathematics, vol. 52 (1930), pp. 617–634.

[3] J. K. Senior and A. C. Lunn, *Determination of the groups of orders* 101–161, *omitting order* 128, American Journal of Mathematics, vol. 56 (1934), pp. 328–338.

[4] M. Potron, *Les groupes d'ordre* p^2, Thesis, Paris, 1904.