

NEW IDENTITIES IN CONFORMAL GEOMETRY

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Complete sets of identities have been obtained for the components of the affine and metric normal tensors and also for the components of the affine, metric, and projective curvature tensors. In addition to these identities, a complete set is known for the components of the first covariant derivative of the affine curvature tensor.¹

In this paper, complete sets of identities are obtained for the components ${}^0B_{jkl}^i$ ($i, j, k, l = 0, 1, \dots, n, \infty$) of the complete conformal curvature tensor² with the exception of the components ${}^0B_{IJ\infty}^0$ and ${}^0B_{\infty J\infty}^I$ ($I, J = 1, \dots, n$). A complete set of identities including the components ${}^0B_{IJ\infty}^0$ and ${}^0B_{\infty J\infty}^I$ has not been written down explicitly because of the excessive complexity of the calculations required. A method of obtaining such identities has been indicated, however, at the end of section 4.

1. Starting with the components³ G_{IJ} of the fundamental conformal tensor defined by

$$G_{IJ} = \frac{g_{IJ}}{|g_{IJ}|^{\frac{1}{n}}},$$

the components K_{BC}^A are defined by⁴

$$(1.1) \quad K_{BC}^A = \frac{1}{2} G^{AS} \left(\frac{\partial G_{BS}}{\partial x^C} + \frac{\partial G_{SC}}{\partial x^B} - \frac{\partial G_{BC}}{\partial x^S} \right),$$

and the K_{BC}^A have the transformation equations⁵

$$(1.2) \quad {}^0C_{AB}^I = \left(K_{MN}^S \frac{\partial x^M}{\partial y^A} \frac{\partial x^N}{\partial y^B} + \frac{\partial^2 x^S}{\partial y^A \partial y^B} \right) \frac{\partial y^I}{\partial x^S} - \frac{1}{n} (\bar{\psi}_A \delta_B^I + \bar{\psi}_B \delta_A^I - \bar{G}_{AB} \bar{G}^{SI} \bar{\psi}_S)$$

under the transformation of coördinates

$$(1.3) \quad x^A = f^A(y^1, \dots, y^n).$$

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¹ T. Y. Thomas, *Differential Invariants of Generalized Spaces*, Cambridge University Press, 1934, pp. 81, 114, 132, 138. This reference will be called T.

² T., p. 81.

³ In this section and the next three, indices have the following ranges: small Latin 0, 1, 2, \dots , n, ∞ ; capital Latin 1, 2, \dots , n ; Greek 0, 1, 2, \dots , n .

⁴ These components K_{BC}^A were originally found in a different form by J. M. Thomas, Proc. Nat. Acad. Sci., vol. 11 (1925), pp. 257-9. They were obtained in the form shown by T. Y. Thomas as indicated in T., p. 67.

⁵ T., p. 68.