ON PROPERTIES OF REGIONS WHICH PERSIST IN THE SUBREGIONS BOUNDED BY LEVEL CURVES OF THE GREEN'S FUNCTION

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1. Let the unit circle $|z| < 1$, which we shall call Q, be mapped by

$$w = f(z), \qquad f(0) = 0,$$

in a one-to-one and conformal manner on a region S in the w-plane. Let S_r be the map of $|z| < r < 1$, the circle Q_r.

The regions S_r have been extensively cultivated. It is known that if S is a convex region, then S_r is convex also. The simplest proof of this is due to Radó.[1] If S is star-shaped with respect to the origin, the like is true of S_r.[2]

These results raise the question of more general properties of S which hold in the subregions S_r. A generalization which includes the properties just mentioned is given here. The method of proof is suggested by Radó's paper.

2. **The property** T. Let $T(w_1, w_2, \cdots, w_n)$ be analytic in $w_1, w_2, \cdots, w_n$ when these variables range over S, and let $T(0, 0, \cdots, 0) = 0$. We shall say that S has the property T if when $w_1, w_2, \cdots, w_n$ lie in S so also does w_0, where

$$w_0 = T(w_1, w_2, \cdots, w_n).$$

As an example, S is convex if any point w_0 on the line segment joining any two points w_1 and w_2 of S is in S:

$$w_0 = T(w_1, w_2) = tw_1 + (1 - t)w_2, \qquad 0 < t < 1.$$

Again, S is star-shaped from the origin if any point w_0 on the line segment joining the origin to any point w_1 of S is in S:

$$w_0 = T(w_1) = tw_1, \qquad 0 < t < 1.$$

Some of the simplest functions T define properties that have not been studied and lead to interesting regions. Consider $T(w_1) = \frac{1}{2}w_1$. S has the property T if the midpoint of the line segment joining the origin to any point of S lies in S. An instructive region with this property is what remains of the unit circle

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[1] T. Radó, *Bemerkung über die konformen Abbildungen konvexer Gebiete*, Math. Ann., vol. 102 (1930), pp. 428–429. The theorem goes back to E. Study, *Konforme Abbildung einfach-zusammenhängender Bereiche*, Leipzig, 1913, p. 110.

[2] W. Seidel, *Über die Ränderzuordnung bei konformen Abbildung*, Math. Ann., vol. 104 (1931), p. 204.