ON SOME CHARACTERIZATIONS OF 2-DIMENSIONAL MANIFOLDS

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1.1. **Object.** A large number of papers have been devoted to the problem of finding topological characterizations for 2-sphere, 2-cell or 2-dimensional manifolds (finite or infinite) of different type. Through complicated cross-citation on the one side, improvements in the available methods on the other side, the study of these papers seems to be at present so much harder than necessary for anybody not already thoroughly acquainted with the ideas used, that the publication of a systematic, simplified exposé of the attained results seems to be the only way of giving these results the place they deserve in the theory of point sets. The results could of course be simplified and extended in several directions. In an appendix we prove that the 2-dimensional generalized manifolds of Čech and Lefschetz are ordinary manifolds.

1.2. **Outline of contents and methods.** The Theorems I to V″ of this paper are very closely related in formulation and proof. This formulation can be reduced to the following scheme. A compact or locally compact Peano space contains at least one curve of one of a few simple types; every curve of that type separates and no closed subset of such a curve separates the space; then the space is homeomorphic with some type of 2-dimensional manifold. In this way we treat in I and III the 2-sphere, in II the closed 2-cell, in IV the 2-dimensional manifold without boundary, in V the open (infinite) 2-dimensional manifolds. The investigation of the set of conditions in IV for a 2-dimensional manifold without boundary was suggested by Zippin. We could have given a characterization of the 2-dimensional manifolds with boundaries by suitably combining the conditions of II and IV. As the result is less elegant and its formulation and proof do not need any additional idea, we leave this to the reader.

The proofs show of course the effect of the similarity in statement. In later proofs many arguments have been left out simply because they have already occurred before. In the proof of I the most important part is the cutting up of the space by a linear graph in arbitrarily small pieces. Different ways of doing this have been used by Moore, Gawehn, Radó (3). We finally used directly a method of approximating a sum of arcs by a linear graph suggested originally by Zippin for the proof of a lemma. The whole argument has been formulated in such a way that it can be used without any change several times more.

Theorems VI and VII contain characterizations by means of Vietoris chains of the compact types of 2-dimensional manifolds. Theorem VII was proved by

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