RIEMANNIAN MANIFOLDS IN THE LARGE

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1. Introduction. The general problem to be considered here is that of determining relations between the local properties of an analytic Riemannian manifold and the topological properties of the manifold in the large. In particular, we are interested in determining topological properties from a knowledge of local properties in the neighborhood of just one point, and conversely, in determining possibilities of metrisation of a given topological manifold by means of local analytic Riemannian geometries.

By an n-dimensional analytic Riemannian manifold in the large will be meant a complete manifold in a sense to be defined later, equivalent to the "normal" Riemannian space of Cartan[1] and a generalization to n dimensions of the "complete surface" of Hopf and Rinow.[2]

The results obtained here are in most cases generalizations of theorems given in the two-dimensional case by Hopf and Rinow. A complete summary of known results on the general problems stated above, as well as a statement of some specific unsolved problems can be found in a paper by Hopf entitled *Differentialgeometrie und topologische Gestalt*, Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 41 (1932), pp. 209–229. Some of the problems proposed there (p. 222, lines 14–22, and p. 224, lines 25–29) are solved in the present paper.

In §2 we define *complete manifolds.* §3 contains a proof that a complete manifold whose Riemannian curvature at every point and with respect to every plane direction is greater than a positive constant e is compact and has a diameter less than $\pi/e^{\frac{1}{2}}$. In §4 is proved the fundamental uniqueness theorem—a given n-dimensional Riemannian element E can be continued to (i.e., contained in) at most one complete, simply connected n-dimensional manifold. In §5 we set up certain coördinate systems in E and give necessary analyticity conditions that E may be continued to a complete manifold. We also show how to determine from a certain coördinate system in the element E about a point A the points conjugate to A. In §6 we show that under certain analyticity conditions an element E about a point A without conjugate point can be continued to a complete manifold homeomorphic to n-dimensional space S_n, from which follows that if a complete simply connected n-dimensional manifold contains a point without conjugate point, it is homeomorphic to S_n. Finally in §7 we

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[1] E. Cartan, *Leçons sur la géométrie des espaces de Riemann*, Paris, 1928.

[2] H. Hopf and W. Rinow, *Über den Begriff der vollständigen differentialgeometrischen Fläche*, Commentarii Mathematici Helvetici, vol. 3 (1931).