AUTOMORPHISMS OF THE PANTS COMPLEX

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In loving memory of my mother, Batya

1. Introduction

In the theory of mapping class groups, "curve complexes" assume a role similar to the one that buildings play in the theory of linear groups. Ivanov, Korkmaz, and Luo showed that the automorphism group of the curve complex for a surface is generally isomorphic to the extended mapping class group of the surface. In this paper, we show that the same is true for the pants complex.

Throughout, S is an orientable surface whose Euler characteristic $\chi(S)$ is negative, while $\Sigma_{g,b}$ denotes a surface of genus g with b boundary components. Also, $\mathrm{Mod}(S)$ means the *extended mapping class group* of S (the group of homotopy classes of self-homeomorphisms of S).

The pants complex of S, denoted $C_P(S)$, has vertices representing pants decompositions of S, edges connecting vertices whose pants decompositions differ by an elementary move, and 2-cells representing certain relations between elementary moves (see Sec. 2). Its 1-skeleton $C_P^1(S)$ is called the *pants graph* and was introduced by Hatcher and Thurston. We give a detailed definition of the pants complex in Section 2.

Brock proved that $C_P^1(S)$ models the Teichmüller space endowed with the Weil-Petersson metric, $\mathscr{T}_{WP}(S)$, in that the spaces are quasi-isometric (see [1]). Our results further indicate that $C_P^1(S)$ is the "right" combinatorial model for $\mathscr{T}_{WP}(S)$, in that $\mathrm{Aut}\, C_P^1(S)$ (the group of simplicial automorphisms of $C_P^1(S)$) is shown to be $\mathrm{Mod}(S)$. This is in consonance with the result of Masur and Wolf that the isometry group of $\mathscr{T}_{WP}(S)$ is $\mathrm{Mod}(S)$ (see [10]).

There is a natural action of $\mathrm{Mod}(S)$ on $C_P^1(S)$; we prove that *all* automorphisms of $C_P^1(S)$ are induced by $\mathrm{Mod}(S)$. The results of this paper can be summarized as follows:

$$\mathrm{Aut}\, C_P(S) \cong \mathrm{Aut}\, C_P^1(S) \cong \mathrm{Mod}(S)$$

for most surfaces S.

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