

# ARITHMETIC OF LINEAR ALGEBRAIC GROUPS OVER 2-DIMENSIONAL GEOMETRIC FIELDS

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*To M. S. Raghunathan on his sixtieth birthday*

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## 0. Introduction

Let  $K$  be the field of fractions of a 2-dimensional, excellent, Henselian, local domain with algebraically closed residue field of characteristic zero. In this introduction, we refer to such a field as a 2-dimensional, strictly Henselian field. (Indeed, a Henselian local ring with a separably closed residue field is called a strictly Henselian local ring.) An example of such a field is the field of fractions  $\mathbf{C}((X, Y))$  of the formal power series ring  $\mathbf{C}[[X, Y]]$ .

In [COP], Ojanguren, Colliot-Thélène, and Parimala investigated quadratic forms over such a field  $K$  and established properties analogous to those familiar over number fields. An analogue of the local-global principle for the Brauer group (going back to work of M. Artin) was also established.

In this paper, we show that most well-known properties of linear algebraic groups

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