

SELMER GROUPS AND THE EISENSTEIN-KLINGEN IDEAL

ERIC URBAN

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Abstract

In this paper, we set up a strategy to prove one divisibility toward the main Iwasawa conjecture for the Selmer groups attached to the twisted adjoint modular Galois representations associated to Hida families. This conjecture asserts the equality of the p -adic L -function interpolating the critical values of the symmetric square of the modular forms in these families and the characteristic ideal of the associated Selmer group. The idea is to introduce a third characteristic ideal containing information on the congruences between cuspidal Siegel modular forms of genus 2 and the Klingen-type Eisenstein series and to prove the two divisibilities: the p -adic L -function divides the Eisenstein ideal, and the Eisenstein ideal divides the characteris-

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