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CLASSIFICATION OF POSITIVE DEFINITE LATTICES

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Contents

1. Classification of positive norm vectors. In this paper we describe an algorithm for classifying orbits of vectors in Lorentzian lattices. The main point of this is that isomorphism classes of positive definite lattices in some genus often correspond to orbits of vectors in some Lorentzian lattice, so we can classify some positive definite lattices. Section 1 gives an overview of this algorithm, and in Section 2 we describe this algorithm more precisely for the case of $II_{1,25}$, and as an application we give the classification of the 665 25-dimensional unimodular positive definite lattices and the 121 even 25-dimensional positive definite lattices of determinant 2 (see Tables 1 and 2). In Section 3 we use this algorithm to show that there is a unique 26-dimensional unimodular positive definite lattice with no roots. Most of the results of this paper are taken from the unpublished manuscript [B4], which contains more details and examples. For general facts about lattices used in this paper, see [CS2, especially Chapters 15–18 and 23–28].

Some previous enumerations of unimodular lattices include Kneser's list of the unimodular lattices of dimension at most 16 [K], Conway and Sloane's extension of this to dimensions at most 23 [CS2, Chapter 16], and Niemeier's enumeration of the even 24-dimensional ones [N]. All of these used some variation of Kneser's neighborhood method [K], but this becomes very hard to use for odd lattices of dimension 24, and it seems impractical for dimension at least 25 (at least for hand calculations; computers could probably push this further). The method used in this paper works well up to 25 dimensions, could be pushed to work for 26 dimensions, and does not seem to work at all beyond this.

We use the "$(+, -, -, \cdots, -)$" sign convention for Lorentzian lattices L, so that the reflections we are interested in are (usually) those of *negative* norm vectors of L.

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