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FROBENIUS$_\infty$ INVARIANTS OF HOMOTOPY GERSTENHABER ALGEBRAS, I

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1. Introduction. Frobenius manifolds play a central role in the usual formulation of mirror symmetry, as may be seen in the following diagram,

A category of compact symplectic manifolds	$\xrightarrow{\text{GW}}$	A category of Frobenius manifolds	$\xleftarrow{\text{BK}}$	A category of compact Calabi-Yau manifolds

where morphisms in all categories are just diffeomorphisms preserving relevant structures, and GW and BK stand, respectively, for the Gromov-Witten (see, e.g., [Ma1]) and Barannikov-Kontsevich (see [BK] and [Ba]) functors. A pair $(\widetilde{M}, M)$ consisting of a symplectic manifold $\widetilde{M}$ and a Calabi-Yau manifold M is said to be *mirror* if $\text{GW}(\widetilde{M}) = \text{BK}(M)$. According to Kontsevich [Ko1], this equivalence is a shadow of a more fundamental equivalence of natural A_∞-categories attached to M and $\widetilde{M}$.

This paper is much motivated by the Barannikov-Kontsevich construction (see [BK] and [Ba]) of the functor from the right in the above diagram, and by Manin's comments [Ma2] on their construction. The roots of the BK functor lie in the extended deformation theory of complex structures on M, more precisely in very special properties of the (differential) Gerstenhaber algebra $\mathfrak{g}$ "controlling" such deformations. One of the miracle features of Calabi-Yau manifolds, the one that played a key role in the BK construction, is that deformations of their complex structures are nonobstructed, always producing a *smooth* versal moduli space.[1]In the language of Gerstenhaber algebras, the exceptional algebraic properties necessary to produce a Frobenius manifold out of $\mathfrak{g}$ have been axiomatized in [Ma1] and [Ma2]. As a result, the functor

"Exceptional" Gerstenhaber algebras	$\xrightarrow{\text{BK}}$	Frobenius manifolds

is now well understood.

[1]A similar phenomenon occurs in the extended deformation theory of Lefschetz symplectic structures which also produces, via the same BK functor, Frobenius manifolds (see [Me1]). These should not be confused with GW.

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