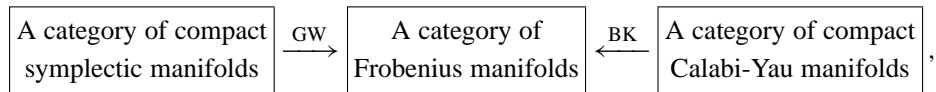


FROBENIUS_∞ INVARIANTS OF HOMOTOPY GERSTENHABER ALGEBRAS, I

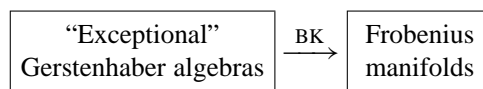
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1. Introduction. Frobenius manifolds play a central role in the usual formulation of mirror symmetry, as may be seen in the following diagram,



where morphisms in all categories are just diffeomorphisms preserving relevant structures, and GW and BK stand, respectively, for the Gromov-Witten (see, e.g., [Ma1]) and Barannikov-Kontsevich (see [BK] and [Ba]) functors. A pair (\tilde{M}, M) consisting of a symplectic manifold \tilde{M} and a Calabi-Yau manifold M is said to be *mirror* if $\text{GW}(\tilde{M}) = \text{BK}(M)$. According to Kontsevich [Ko1], this equivalence is a shadow of a more fundamental equivalence of natural A_∞ -categories attached to M and \tilde{M} .

This paper is much motivated by the Barannikov-Kontsevich construction (see [BK] and [Ba]) of the functor from the right in the above diagram, and by Manin's comments [Ma2] on their construction. The roots of the BK functor lie in the extended deformation theory of complex structures on M , more precisely in very special properties of the (differential) Gerstenhaber algebra \mathfrak{g} "controlling" such deformations. One of the miracle features of Calabi-Yau manifolds, the one that played a key role in the BK construction, is that deformations of their complex structures are nonobstructed, always producing a *smooth* versal moduli space.¹ In the language of Gerstenhaber algebras, the exceptional algebraic properties necessary to produce a Frobenius manifold out of \mathfrak{g} have been axiomatized in [Ma1] and [Ma2]. As a result, the functor



is now well understood.

¹A similar phenomenon occurs in the extended deformation theory of Lefschetz symplectic structures which also produces, via the same BK functor, Frobenius manifolds (see [Me1]). These should not be confused with GW.

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