UNIQUENESS FOR LOCALLY INTEGRABLE SOLUTIONS OF OVERDETERMINED SYSTEMS

S. BERHANU AND J. HOUNIE

0. Introduction. It is well known that if $u = u(z_1, \dots, z_n)$ is a holomorphic function defined on a connected open set in $\mathbb{C}^n$ and vanishes on a subset of positive measure, then it vanishes everywhere. A holomorphic function may be regarded as a solution of the overdetermined system of equations

$$\frac{\partial u}{\partial \bar{z}_j} = 0, \quad j = 1, \dots, n.$$

In this paper we explore generalizations of this property for locally integrable solutions of overdetermined systems

$$L_j u = 0, \quad j = 1, \dots, n, \tag{0.1}$$

where the L_j are linearly independent vector fields with smooth, complex-valued coefficients defined on some open region in $\mathbb{R}^N$. In contrast to the case of the Cauchy-Riemann (CR) equations, a locally integrable solution $u(x_1, \dots, x_N)$ of the system of $n \le N-1$ equations

$$\frac{\partial u}{\partial x_j} = 0, \quad j = 1, \dots, n,$$

may very well vanish on a set of positive measure without vanishing identically. Here we explain these different behaviors in terms of geometric objects associated to each system, namely, the family of orbits of the system (0.1) (see Section 3 for precise definitions). Since the properties involved do not change if each vector L_j in (0.1) is replaced by a vector $L'_j = \sum_k a_{jk}(x) L_k$ where the smooth matrix $[a_{jk}(x)]$ is invertible, we state our results in the framework of a structure $\mathscr{L}$ of rank-n defined on an open set Ω in $\mathbb{R}^N$ of which the vectors (0.1) are local generators. Most of the time we assume that the structure is locally integrable, that is, that every point of Ω has a neighborhood where a set of $m = N - n$ first integrals $Z_1, \dots, Z_m$ of the structure (i.e., solutions of $\mathscr{L} Z_j = 0$) are defined and satisfy $dZ_1 \wedge \cdots \wedge dZ_m \ne 0$. On the subject of locally integrable structures, we refer to [T]. One of our results is the following theorem.

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