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QUANTUM DEFORMATION OF WHITTAKER MODULES AND THE TODA LATTICE

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Introduction. In 1978 Kostant suggested the *Whittaker model* of the center of the universal enveloping algebra $U(\mathfrak{g})$ of a complex simple Lie algebra $\mathfrak{g}$. An essential role in this construction is played by a nonsingular character χ of the maximal nilpotent subalgebra $\mathfrak{n}_+ \subset \mathfrak{g}$. The main result is that the center of $U(\mathfrak{g})$ is isomorphic to a commutative subalgebra in $U(\mathfrak{b}_-)$, where $\mathfrak{b}_- \subset \mathfrak{g}$ is the opposite Borel subalgebra. This observation is used in the theory of principal series representations of the corresponding Lie group G and in the proof of complete integrability of the quantum Toda lattice.

The goal of this paper is to generalize Kostant's construction to quantum groups. An obvious obstruction is the fact that the subalgebra in $U_h(\mathfrak{g})$ generated by positive root generators (subject to the quantum Serre relations) does not have nonsingular characters. In order to overcome this difficulty we use a family of new realizations of quantum groups introduced in [13]. The modified quantum Serre relations allow for nonsingular characters, and we are able to construct the Whittaker model of the center of $U_h(\mathfrak{g})$.

Using the Whittaker model of the center of $U_h(\mathfrak{g})$, we introduce quantum deformations of Whittaker modules. The new Whittaker model is also applied to the deformed quantum Toda lattice recently studied by Etingof [6]. We give new proofs of his results which resemble the original Kostant's proofs for the quantum Toda lattice.

The paper is organized as follows. Section 1 contains a review of Kostant's results on the Whittaker model and Whittaker modules [11], [10]. In order to create a pattern for proofs in the quantum group case, we recall most of Kostant's proofs. The central part of the paper is Section 2. There we discuss properties of new realizations of finite-dimensional quantum groups and present the Whittaker model of the center of $U_h(\mathfrak{g})$. In Section 2.4 we introduce quantum deformed Whittaker modules. Section 2.6 contains a discussion of the deformed quantum Toda lattice.

1. Whittaker modules.

In this section we recall the Whittaker model of the center of the universal enveloping algebra $U(\mathfrak{g})$, where $\mathfrak{g}$ is a complex simple Lie algebra.

1.1. Notation. Fix the notation used throughout the text. Let G be a connected simply connected finite-dimensional complex simple Lie group, and let $\mathfrak{g}$ be its Lie algebra. Fix a Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$, and let Δ be the set of roots of $(\mathfrak{g}, \mathfrak{h})$. Choose

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