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RATIONAL POINTS ON QUARTICS

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CONTENTS

1. Introduction. Of all the possible extensions to higher dimensions of Faltings's theorem, probably the most fundamental is the following conjecture.

CONJECTURE 1.1 (Weak Lang conjecture). *Let X be a variety defined over a number field K. If X is of general type, then the set $X(K)$ of K-rational points of X is not Zariski dense.*

(While the name "weak Lang conjecture" has become standard usage—in part to distinguish it from the "strong Lang conjecture" below—we should point out that as stated here it was first ventured by Bombieri for surfaces (see, e.g., [28]) and by Vojta in [32].)

We ask now whether a converse to this statement might hold. As it stands, the converse to the weak Lang conjecture cannot possibly be true. For example, if we take the product $X = \mathbb{P}^1 \times C$ of a rational curve and a curve C of genus $g \geq 2$, we get a surface that is not of general type; but by Faltings's theorem the rational points of X must lie in a finite union of fibers of X over C.

The point is that the Kodaira dimension of a variety is not a sufficiently sensitive measure of the positivity or negativity of its canonical bundle. One possible modification, if we hope to have a plausible converse to the weak Lang conjecture, is to

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