Vol. 104, No. 3 DUKE MATHEMATICAL JOURNAL

ON THE FINITE-GAP ANSATZ IN THE CONTINUUM LIMIT OF THE TODA LATTICE

A. B. J. KUIJLAARS

1. Introduction. The finite, nonperiodic Toda lattice is a dynamical system given by the Lax pair $dL/dt = BL - LB$ for tridiagonal $n \times n$ matrices

$$L = \begin{bmatrix} a_1 & b_1 & & & 0 \\ b_1 & a_2 & b_2 & & \\ & b_2 & a_3 & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ 0 & & & b_{n-1} & a_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & b_1 & & & 0 \\ -b_1 & 0 & b_2 & & \\ & -b_2 & 0 & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ 0 & & & -b_{n-1} & 0 \end{bmatrix}.$$

This corresponds to the system of equations

$$\frac{da_k}{dt} = 2\left(b_k^2 - b_{k-1}^2\right), \quad k = 1, \ldots, n, \tag{1.1}$$

$$\frac{db_k}{dt} = b_k(a_{k+1} - a_k), \quad k = 1, \ldots, n-1, \tag{1.2}$$

with $b_0 = b_n = 0$. The Toda lattice is an integrable system that is solved explicitly by the inverse spectral method (see [M]).

Deift and McLaughlin [DM] studied the continuum limit of the Toda lattice. Here the size n tends to infinity, and we write $a_k(t;n)$ and $b_k(t;n)$ to indicate the dependence on n. For given continuous functions $a_0(y)$ and $b_0(y) > 0$ for $y \in (0,1)$, Deift and McLaughlin [DM] take initial values

$$a_k(0;n) = a_0\left(\frac{k}{n}\right), \qquad b_k(0;n) = b_0\left(\frac{k}{n}\right) \tag{1.3}$$

and study the limiting behavior of

$$a_{[xn]}(tn;n), \qquad b_{[xn]}(tn;n)$$

as $n \to \infty$ with fixed $x \in (0,1)$ and $t > 0$.

Received 7 June 1999.
2000 *Mathematics Subject Classification*. Primary 37J35, 37K10; Secondary 31A15, 35Q53.
Author's work supported in part by research project number G.0278.97 and by a research grant from the Fonds voor Wetenschappelijk Onderzoek–Vlaanderen (FWO).