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# REFLECTION GROUPS OF LORENTZIAN LATTICES

RICHARD E. BORCHERDS

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**0. Introduction.** The aim of this paper is to provide evidence for the following new principle: Interesting reflection groups of Lorentzian lattices are controlled by certain modular forms with poles at cusps. We use this principle to explain many of the known examples of such reflection groups and to find several new examples of reflection groups of Lorentzian lattices, including one whose fundamental domain has 960 faces.

We do not give a precise definition of what it means for a reflection group of a Lorentzian lattice to be interesting, mainly because there seem to be occasional counterexamples to almost any precise version of the principle. However, the interesting groups should include the cases when the reflection group is cofinite or, more generally, when the quotient of the full automorphism group by the reflection group contains a free abelian subgroup of finite index.

The main idea of this paper is roughly as follows (and is described in more detail in Section 11). Suppose that $L$ is an even level $N$ lattice in $\mathbf{R}^{1,n}$. Then the idea is that if

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