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MONODROMY OF AIRY AND KLOOSTERMAN SHEAVES

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1. Introduction. Our work began with calculations of the numbers of points on a family of hyperelliptic curves given by the following equation over a finite field k of characteristic 2:

$$y^2 - y = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$$

The simplicity of this equation makes these hyperelliptic curves particularly well suited for explicit computer calculations requiring higher genus curves. On the other hand, the special form of the equation raises the question of how generic or random, if at all, these curves are. For instance, all of these curves have 2-rank 0.

The question of genericity can be restated more precisely as the question of the monodromy group of a family of curves. Let us denote by K the field $k(a_i)$, where a_i are the variable parameters of a family C of hyperelliptic curves. Choosing $l \neq 2$, the l-adic monodromy group is the image of the action of $\mathrm{Gal}(K^{\mathrm{sep}}/K)$ on $H^1(C \otimes_K \bar{K}, \mathbf{Q}_l)$. Many properties of a family C of curves are described by the geometric monodromy group G_{geom}, which is the Zariski closure of image of the subgroup $\mathrm{Gal}(K^{\mathrm{sep}}/K\bar{k})$. For instance, if the geometric monodromy group of a family C is finite, all curves in the family C are supersingular. We prove that quite the opposite is true.

THEOREM (Corollary 13.3). *For any $g \geq 3$, the geometric monodromy group of hyperelliptic curves of genus g and 2-rank zero over $\overline{\mathbf{F}}_2$ is* $\mathrm{Sp}(2g)$.

Of several possible approaches to studying the monodromy group of the above family, we chose the one using Fourier transforms. In naive terms, a Fourier transform corresponds to a varying coefficient a_1 of the family over values in $\bar{k}$. From a higher viewpoint, the cohomology sheaf $\mathscr{F}$ of this subfamily is the Fourier transform of a rank-1 $\overline{\mathbf{Q}}_l$ sheaf lisse on the affine line over $\bar{k}$. General theory of Fourier transforms guarantees that irreducibles transform into irreducibles, and therefore the geometric monodromy group of this subfamily is always irreducible.

Another key restriction on the sheaf $\mathscr{F}$ arises from the action of the inertia group. Again, the general theory of local Fourier transforms guarantees that the inertia group at ∞ of $\mathbf{A}^1$ acts irreducibly. It is, however, hard to describe this action, because even the action of wild inertia does not factor via an abelian quotient. Still, it is possible to show that the sheaf of traceless endomorphisms $\mathrm{End}^0(\mathscr{F})$ has no factors of dimension less than $\mathscr{F}$. This property of $\mathscr{F}$ is used to derive strong restrictions on its G_{geom} (see

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