Vol. 103, No. 3 DUKE MATHEMATICAL JOURNAL © 2000

ONE-DIMENSIONAL SYMMETRY OF BOUNDED ENTIRE SOLUTIONS OF SOME ELLIPTIC EQUATIONS

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1. Introduction. This article is devoted to the classification of the functions u that are solutions of the semilinear elliptic equation

$$\Delta u + f(u) = 0 \quad \text{in } \mathbb{R}^n \tag{1.1}$$

and that satisfy $|u| \le 1$ together with the asymptotic conditions

$$u(x', x_n) \xrightarrow[x_n \to \pm\infty]{} \pm 1 \quad \text{uniformly in } x' = (x_1, \dots, x_{n-1}). \tag{1.2}$$

The given function $f = f(u)$ is Lipschitz-continuous in $[-1, 1]$. Clearly, for (1.1), (1.2) to have a solution, f has to be such that $f(\pm 1) = 0$. Here we assume furthermore that there exists $\delta > 0$ such that

$$f \text{ is nonincreasing on } [-1, -1+\delta] \text{ and on } [1-\delta, 1]; \qquad f(\pm 1) = 0. \tag{1.3}$$

We prove that any solution u of the multidimensional equation (1.1) with the limiting conditions (1.2) has one-dimensional symmetry.

THEOREM 1. *Let u be a solution of (1.1), (1.2) such that $|u| \le 1$. Then $u(x', x_n) = u_0(x_n)$, where u_0 is a solution of*

$$\begin{cases} u_0'' + f(u_0) = 0 & \text{in } \mathbb{R}, \\ u_0(\pm\infty) = \pm 1, \end{cases} \tag{1.4}$$

and u is increasing with respect to x_n. In particular, the existence of a solution u of (1.1), (1.2) such that $|u| \le 1$ implies the existence of a solution u_0 of (1.4). Lastly, this solution u is unique up to translations of the origin.

For the 1-dimensional problem, we refer to [5], [11], [19], or [23]. For the low dimensions case $n = 2, 3$ (assuming also that f is C^1), the same result had been obtained by Ghoussoub and Gui [21]. Their method relies on spectral properties of some Schrödinger operators and is different from the one we use in this paper in any dimension n. We have recently learned that a result similar to Theorem 1 has been proved independently by Barlow, Bass, and Gui [7], using a very different method relying on probabilistic arguments.

Let us point out that Theorem 1 is related to a more difficult question, known as a conjecture of De Giorgi.

Received 17 February 1999.
2000 *Mathematics Subject Classification*. Primary 35J60; Secondary 35B05, 35B40, 35B50.