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ANALYTIC STRATIFICATION IN THE PFAFFIAN CLOSURE OF AN O-MINIMAL STRUCTURE

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Introduction. Let $U \subseteq \mathbb{R}^n$ be open and $\omega = a_1 dx_1 + \cdots + a_n dx_n$ a nonsingular, integrable 1-form on U of class C^1, and let $\mathcal{F}$ be the foliation on U associated to ω. A leaf $L \subseteq U$ of $\mathcal{F}$ is a *Rolle leaf* if any C^1 curve $\gamma : [0,1] \to U$ with $\gamma(0), \gamma(1) \in L$ is tangent to $\mathcal{F}$ at some point, that is, $\omega(\gamma(t))(\gamma'(t)) = 0$ for some $t \in [0,1]$. Note that while a leaf of $\mathcal{F}$ is in general only an immersed manifold, any Rolle leaf of $\mathcal{F}$ is an embedded and closed submanifold of U.

Throughout this paper, we fix an arbitrary o-minimal expansion $\widetilde{\mathbb{R}}$ of the field of real numbers. Whenever U and $a_1, \dots, a_n$ are definable in $\widetilde{\mathbb{R}}$, then a leaf of $\mathcal{F}$ is called a leaf *over* $\widetilde{\mathbb{R}}$. We use $\widetilde{\mathbb{R}}_1$ to denote the expansion of $\widetilde{\mathbb{R}}$ by all Rolle leaves over $\widetilde{\mathbb{R}}$.

For example, the expansion $\mathbb{R}_{\text{an}}$ of the real field generated by all globally semianalytic sets is o-minimal; in fact the sets definable in $\mathbb{R}_{\text{an}}$ are exactly the globally subanalytic sets (see [7], [4]). Building on Khovanskiĭ's theory of fewnomials [10] and subsequent work by Moussu and Roche [14], Lion and Rolin [12] showed that $(\mathbb{R}_{\text{an}})_1$ is also o-minimal. Adapting the various ideas involved to the general o-minimal setting, Speissegger [15] proved the following statement.

FACT. *The structure $\widetilde{\mathbb{R}}_1$ is o-minimal.*

The o-minimal structure $\widetilde{\mathbb{R}}$ is said to *admit analytic cell decomposition* if, for any finite collection $A_1, \dots, A_k \subseteq \mathbb{R}^n$ of sets definable in $\widetilde{\mathbb{R}}$, there is a decomposition Γ of $\mathbb{R}^n$ into finitely many analytic cells definable in $\widetilde{\mathbb{R}}$, such that each A_i is a union of cells in Γ. In this paper we establish the following statement.

THEOREM. *If $\widetilde{\mathbb{R}}$ admits analytic cell decomposition, then so does $\widetilde{\mathbb{R}}_1$.*

We assume that the reader is familiar with the terminology introduced in [6] (for instance, "C^k cell," "cell decomposition," "Whitney stratification," etc.). By the general results on o-minimal expansions of the real field described there, the theorem can be restated as follows, thereby generalizing the results obtained by Cano, Lion and Moussu in [3].

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