DISTRIBUTION OF ALMOST DIVISION POINTS

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1. Introduction. In [10], we proved an equidistribution theorem for small points on abelian varieties, based on the ideas in [7] and [8]. In this paper, we want to generalize this result to *almost division points*. In the following, we describe our main theorem and its application to the discreteness of almost division points on subvarieties.

Let A be an abelian variety defined over a number field K. Let x_n ($n \in \mathbb{N}$) be a sequence of distinct points in $A(\bar{K})$. We assume this is a sequence of almost division points, which means

$$\lim_{n\to\infty} \sup_{\sigma\in G} \left\| x_n^\sigma - x_n \right\| = 0.$$

Here, $G = \mathrm{Gal}(\bar{K}/K)$, and $\|\cdot\|$ is the square root of the Neron-Tate height function, with respect to some ample and symmetric line bundle on A. Obviously, the notion of almost division does not depend on the choice of the Neron-Tate height functions. If we drop the limit in the above equality, then all x_n are division points for $A(K)$.

We fix an embedding $\sigma : \bar{K} \to \mathbb{C}$; then $A(\bar{K})$ can be considered a subgroup of $A(\mathbb{C}) := A_\sigma(\mathbb{C})$. The Galois orbits x_n^G therefore define a sequence δx_n^G of probability measures on $A(\mathbb{C})$; if f is a continuous function on $A(\mathbb{C})$, then

$$\int_{A(\mathbb{C})} f \delta x_n^G = \frac{1}{\left|x_n^G\right|} \sum_{y\in x_n^G} f(y).$$

In this paper, we address the convergence of δx_n^G. More precisely, we want to know whether there is a measure $d\mu$ on $A(\mathbb{C})$ such that, for any continuous function f on $A(\mathbb{C})$,

$$\lim_{n\to\infty} \int_{A(\mathbb{C})} f\, dx_n^G = \int_{A(\mathbb{C})} f\, d\mu.$$

Obviously, such a measure $d\mu$ does not exist in general; but, since the space of the continuous functions on $A(\mathbb{C})$ can be topologically generated by countably many functions, $d\mu$ does exist if $(x_n, n \in \mathbb{N})$ is replaced with a subsequence. So, our purpose becomes to describe the following:

- the property of the sequence $(x_n, n \in \mathbb{N})$ which can be obtained by replacing it with a subsequence;

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