Vol. 102, No. 3 DUKE MATHEMATICAL JOURNAL

ABUNDANCE THEOREM FOR SEMI LOG CANONICAL THREEFOLDS

OSAMU FUJINO

Contents

0. Introduction. The main purpose of this paper is to prove the abundance theorem for semi log canonical threefolds. The abundance conjecture is a very important problem in the birational classification of algebraic varieties. The abundance theorem for semi log canonical surfaces was proved in [1] and [15] by D. Abramovich, L.-Y. Fong, S. Keel, J. Kollár, and J. Mckernan. The proof uses semiresolution, and so on, and has some combinatorial complexities. We simplify the proof and generalize the theorem to semi divisorial log terminal surfaces (see Corollary 4.10). By our method we can reduce the problem to the irreducible case and the finiteness of some groups. This shows that if the log Minimal Model Program (log MMP, for short), the log abundance conjecture for n-folds, and the finiteness of B-pluricanonical representations (see Section 3) hold for $(n-1)$-folds, then the abundance conjecture for semi log canonical n-folds is true almost automatically (see Theorem A.1 in the appendix). But unfortunately the log MMP and the log abundance conjecture are only conjectures for n-folds with $n \geq 4$. So we prove the following theorem.

Theorem 0.1 (Abundance theorem for slc threefolds). *Let (X, Δ) be a proper semi log canonical (slc, for short) threefold with $K_X + \Delta$ nef. Then $K_X + \Delta$ is semiample.*

This theorem is a generalization of the abundance theorem for log canonical threefolds proved by S. Keel, K. Matsuki, and J. McKernan (see [16]). According to the authors, the abundance theorem for log canonical threefolds is considered to be the first step towards a proof of the abundance conjecture in dimension 4. We believe that the abundance theorem for semi log canonical threefolds is the second step.

Received 2 October 1998. Revision received 6 July 1999.
1991 *Mathematics Subject Classification*. Primary 14E30; Secondary 14E07.
Research Fellow of the Japan Society for the Promotion of Science.