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HOLOMORPHIC CYLINDERS WITH LAGRANGIAN BOUNDARIES AND HAMILTONIAN DYNAMICS

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§1. Introduction. Given a closed Lagrangian submanifold L of a symplectic manifold (M,ω), we consider the moduli space of J-holomorphic cylinders u with boundary in L which belong to a given free homotopy class $\alpha \in [C,\partial C; M,L]$. Actually, we are interested in the cases (M,L,α) for which the moduli space of Fredholm-regular unparametrised J-holomorphic cylinders reduces to an isolated family containing an odd number of elements (a single one, for instance). If, say, the manifold contains no holomorphic spheres and if the two boundary components of the cylinder are mapped to two disjoint components L_0, L_1 of L, then an argument similar to the one used by Hofer and Viterbo in [HV2] (or by Gromov in [G]) shows the existence of a periodic orbit of any Hamiltonian that separates L_0 from L_1 (i.e., reaches on L_1 a minimum value larger than its maximum value on L_0), at least when the growth at infinity of H is controlled (in case the manifold M is noncompact) and when holomorphic discs with boundary in L can be avoided. The idea is to perturb the equation

$$du + J \circ du \circ j = 0,$$

where j is any of the conformal structures of the cylinder and where J belongs to the set $\mathcal{J}(M,\omega)$ of ω-compatible almost complex structures on M, by introducing a new term that depends on the given Hamiltonian H. The result then follows from a detailed study of the solutions of the perturbed equation that are in the class α. Essentially, this result tells us that the difference $\min_{L_1} H - \max_{L_0} H$ (or, more generally, $\int_0^1 (\min_{L_1} H_t - \max_{L_0} H_t)\, dt$ in the nonautonomous case) is a kind of capacity whose positivity ensures the existence of periodic orbits.

Note that, because the cylinder $C = [0,1] \times S^1$ is parallelizable, the Cauchy-Riemann equation for maps $u : C \to M$ has a direct interpretation in terms of vector fields on C, and perturbing this equation to introduce the Hamiltonian term is straightforward. Second, because the group of biholomorphisms of C, for any choice of complex structure, is just the compact space S^1, the passage from solutions of the Cauchy-Riemann equation to unparametrised J-curves is simple. As we will see, an important property of the perturbed equation is naturally expressed in terms of the

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