

# FORMAL CHOW GROUPS, $p$ -DIVISIBLE GROUPS, AND SYNTOMIC COHOMOLOGY

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**1. Introduction.** Let  $K$  be an absolutely unramified complete discrete valuation field of mixed characteristic  $(0, p)$  with perfect residue field  $k$ , and  $V$  a smooth projective variety over  $K$ . In this article, we consider the Chow groups  $\mathrm{CH}^r(V)$  in the infinitesimal method, which was proposed by Spencer Bloch (cf. [5, p. 24]). We review this method in what follows.

First of all, recall the modified version of the Bloch-Quillen formula (cf. [16])

$$\mathrm{CH}^r(V) \otimes \mathbb{Z} \left[ \frac{1}{(r-1)!} \right] \cong H^r(V, \mathcal{K}_{r,V}^M) \otimes \mathbb{Z} \left[ \frac{1}{(r-1)!} \right],$$

where, for any scheme  $Z$ , we denote by  $\mathcal{K}_{r,Z}^M$  the Zariski sheaf of Milnor  $K_r$ -groups on  $Z$ , that is, the sheafification of the presheaf that associates the Milnor  $K$ -group  $K_r^M(\Gamma(U, \mathcal{O}_U))$  to an open subscheme  $U$  of  $Z$ . Here, for any commutative ring  $R$ , we denote by  $K_r^M(R)$  the group  $R^{*\otimes r}/H$ , where  $H$  is the subgroup of  $R^{*\otimes r}$  generated by elements of the form  $x_1 \otimes \cdots \otimes x_r$  with  $x_i + x_j = 0$  or  $1$  for some  $i \neq j$ . If  $r$  is equal to the dimension of  $V$ , the above formula is valid without tensoring  $\mathbb{Z}[1/(r-1)!]$  (cf. [10]).

Now assume that  $V$  admits a smooth projective model  $X$  over the valuation ring  $W$  of  $K$  (i.e.,  $W = W(k)$  is the ring of Witt vectors of  $k$ ). Let  $\mathcal{A}$  be the category of Artinian local  $W$ -algebras that have  $k$  as their residue field. For any object  $A$  of  $\mathcal{A}$ ,

Received 27 April 1999. Revision received 20 September 1999.

1991 *Mathematics Subject Classification*. Primary 14G20; Secondary 11G25, 14F30, 19E20.

Author's research supported by Japan Society for the Promotion of Science (JSPS) Research Fellowships for Young Scientists.