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EXCEPTIONAL INTEGERS FOR GENERA OF INTEGRAL TERNARY POSITIVE DEFINITE QUADRATIC FORMS

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0. Introduction. In [2] it was shown that in a certain sense most integers represented by some form in the genus of a given integral ternary positive definite quadratic form are represented by all forms in this genus. More precisely, let (L, q) be a **Z**-lattice of rank 3 with integral positive definite quadratic form. Then all sufficiently large integers a that are represented primitively by some lattice in the spinor genus of (L, q) are represented by all lattices in that spinor genus (see corollary to Theorem 3 in [2]). The theorems of that article actually imply a slightly sharper characterization of the set of exceptional integers that are represented by some forms in the genus of L but not by all of them. Since there seems to be some interest to have available a characterization of this set that is as sharp as possible, I give such a description and some examples in this note. I also comment on the question of effectivity of the results and on results for primitive representations.

As in [2], a special role is played by the integers t in the square class of a primitive spinor exception, that is, in the square class of an integer represented primitively by some but not by all spinor genera in the genus of (L, q). The Fourier coefficients of the spinor generic theta series at integers tp^2 for primes p in certain arithmetic progressions do not grow for growing p. In view of the positivity of the Fourier coefficients of theta series, this implies that the Shimura lift with respect to such a square class of the difference of the theta series of lattices in the same spinor genus omits these primes in its Fourier expansion. One might therefore be tempted to speculate about a connection to CM-forms. By looking at an example, we see, however, that this is not the case; in general one has to expect that one is looking at the sum of a cusp form and of its quadratic twist.

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1. Exceptional integers. Let (L, q) as above be a quadratic lattice of level N, that is, for the dual lattice $L^\#$, we have $q(L^\#)\mathbf{Z} = N^{-1}\mathbf{Z}$. Let d denote the discriminant of (L, q). Let T denote the (finite) set of primes p for which (L, q) remains anisotropic over the p-adic completion $\mathbf{Q}_p$ (for $p \in T$, we have $p \mid N$) and write $\overline{q}(L)$ for the set of numbers represented by some lattice in the genus of (L, q) (or equivalently, locally everywhere by (L, q)), $\overline{q_r}(L)$ for the set of $t \in \overline{q}(L)$ that are divisible at

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