Vol. 102, No. 1 DUKE MATHEMATICAL JOURNAL

THE GRIFFITHS GROUP OF A GENERAL CALABI-YAU THREEFOLD IS NOT FINITELY GENERATED

CLAIRE VOISIN

1. Introduction. If X is a Kähler variety, the intermediate Jacobian $J^{2k-1}(X)$ is defined as the complex torus

$$J^{2k-1}(X) = H^{2k-1}(X,\mathbb{C})/F^k H^{2k-1}(X) \oplus H^{2k-1}(X,\mathbb{Z}),$$

where $F^k H^{2k-1}(X)$ is the set of classes representable by a closed form in $F^k A^{2k-1}(X)$, that is, which is locally of the form $\sum_{I,J} \alpha_{I,J}\, dz_I \wedge d\bar{z}_J$, with $|I|+|J| = 2k-1$ and $|I| \geq k$.

Griffiths [9] has defined the Abel-Jacobi map

$$\Phi_X^k : \mathcal{Z}^k_{\mathrm{hom}}(X) \longrightarrow J^{2k-1}(X),$$

where $\mathcal{Z}^k_{\mathrm{hom}}(X)$ is the group of codimension k algebraic cycles homologous to zero on X. Using the identification

$$J^{2k-1}(X) = \frac{\left(F^{n-k+1} H^{2n-2k+1}(X)\right)^*}{H_{2n-2k+1}(X,\mathbb{Z})}, \quad n = \dim X$$

given by Poincaré duality, Φ_X^k associates to the cycle $Z = \partial\Gamma$, where Γ is a real chain of dimension $2n-2k+1$ well defined up to a $2n-2k+1$-cycle, the element

$$\int_\Gamma \in \left(F^{n-k+1} H^{2n-2k+1}(X)\right)^* / H_{2n-2k+1}(X,\mathbb{Z}),$$

which is well defined using the isomorphism

$$F^{n-k+1} H^{2n-2k+1}(X) \cong \frac{F^{n-k+1} A^{2n-2k+1}(X)^c}{d F^{n-k+1} A^{2n-2k}(X)}.$$

If $(Z_t)_{t\in C}$ is a flat family of codimension k algebraic cycles on X parametrized by a smooth irreducible curve C, the map $t \mapsto \Phi_X^k(Z_t - Z_0)$ factors through a homomorphism from the Jacobian $J(C)$ to $J^{2k-1}(X)$, and one can show that the image of this morphism is a complex subtorus of $J^{2k-1}(X)$ whose tangent space is contained in $H^{k-1,k}(X) \subset H^{2k-1}(X,\mathbb{C})/F^k H^{2k-1}(X)$. Defining the subgroup

Received 9 March 1999.
1991 *Mathematics Subject Classification*. Primary 14C25, 14C30, 14M99, 32J25.
Author's work partially supported by the project Algebraic Geometry in Europe.