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GROUP ACTIONS ON S^6 AND COMPLEX STRUCTURES ON $\mathbb{P}_3$

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1. Introduction. This note is motivated by the following classical problem: Is there a complex structure on the 6-sphere S^6, in other words, is S^6 a complex manifold? It has been known since the paper [BS] that all other spheres S^{2n}, $n > 1$, do not even admit an almost complex structure. On the other hand S^6 admits many almost complex structures; see the presentation in [Ste, Part III, Sect. 41.17]. It is generally believed that none of them is integrable.

Suppose S^6 has a complex structure X. Then by [CDP] every meromorphic function on X is constant. Moreover X is not Kähler, since $b_2(X) = 0$. Therefore the problem is quite inaccessible by standard methods of complex geometry.

In this paper we prove the following theorem.

THEOREM 1.1. *X is not almost homogeneous. In other words, the automorphism group* $\mathrm{Aut}_{\mathbb{C}}(X)$ *does not have an open orbit.*

This is related as follows to the question of existence of complex structures on the underlying differentiable manifold of $\mathbb{P}_3(\mathbb{C})$: As above, assume $X = S^6$ has the structure of a complex manifold. For $p \in X$, let $\pi_p : X_p \to X$ denote the blow-up of X at p with $\pi_p^{-1}(p) =: E_p$. Of course $E_p = \mathbb{P}_2(\mathbb{C})$. Since sufficiently small neighborhoods of a hyperplane in $\mathbb{P}_n(\mathbb{C})$ are differentiably identifiable with neighborhoods

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