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A TOPOLOGICAL RECONSTRUCTION THEOREM FOR $\mathcal{D}^\infty$-MODULES

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Contents

0. Introduction. In algebraic analysis, one represents systems of analytic linear partial differential equations on a complex analytic manifold X by modules over the ring $\mathcal{D}_X$ of linear partial differential operators with analytic coefficients. Using this representation, the holomorphic solutions of the homogeneous system associated to the $\mathcal{D}_X$-module $\mathcal{M}$ correspond to

$$\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{O}_X),$$

where $\mathcal{O}_X$ denotes the $\mathcal{D}_X$-module of holomorphic functions. If one wants also to take into consideration the compatibility conditions, then one has to study the full solution complex

$$\mathcal{S}ol(\mathcal{M}) = R\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{O}_X)$$

in the derived category $D^+(\mathbb{C}_X)$ of sheaves of $\mathbb{C}$-vector spaces. In [6] (see also [9]), it was shown that the functor $\mathcal{S}ol$ induces an equivalence between the derived category formed by the bounded complexes of regular holonomic $\mathcal{D}_X$-modules and that formed by the bounded complexes of $\mathbb{C}$-constructible $\mathbb{C}_X$-modules. This equivalence is usually called the Riemann-Hilbert correspondence. One of its corollaries is that it is possible to reconstruct a complex of regular holonomic $\mathcal{D}_X$-modules from its complex of holomorphic solutions.

Our aim in this paper is to extend this reconstruction theorem to perfect complexes of $\mathcal{D}_X^\infty$-modules by taking into account the natural topology of the complex of

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