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ON THE SPECTRUM OF CERTAIN DISCRETE SCHRÖDINGER OPERATORS WITH QUASIPERIODIC POTENTIAL

FLORIN P. BOCA

Throughout this paper, we denote $\ell_d^2 = \ell^2(\mathbf{Z}^d)$, $d \geq 1$, with canonical orthonormal basis $\{u(\mathbf{n})\}_{\mathbf{n}}$, and $L_d^2 = L^2(\mathbf{T}^d)$. The torus $\mathbf{T}^d$ is freely identified with $\mathbf{R}^d/\mathbf{Z}^d = [0,1)^d$. We set $\|x\| = \operatorname{dist}(x, \mathbf{Z}^d)$, $x \in \mathbf{R}^d$. The vector in $\mathbf{Z}^d$, whose kth component is zero if $k \neq j$ and one if $k = j$, is denoted by e_j, $1 \leq j \leq d$. Set also $\mathbf{e} = e_1 + \cdots + e_d$ and $\mathbf{0} = (0, \ldots, 0) \in \mathbf{Z}^d$. In the sequel, we often refer to the following two functions:

$$h_z(\theta) = z - \sum_{j=1}^{d} 2\cos 2\pi\theta_j, \quad \theta = (\theta_1, \ldots, \theta_d) \in \mathbf{R}^d, \ z \in \mathbf{C},$$

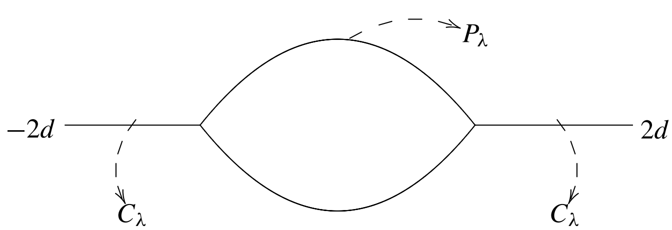
$$G(z) = (2\pi)^{-d} \int_{\mathbf{T}^d} \log|h_z(\theta)|\, d\theta, \quad z \in \mathbf{C} \setminus [-2d, 2d].$$

For each $\lambda > 0$, denote $c_\lambda(z) = G(z) - \log\lambda$ and consider

$$P_\lambda = \{z \in \mathbf{C}; c_\lambda(z) = 0\} \qquad \text{and} \qquad C_\lambda = \{z \in \mathbf{C}; c_\lambda(z) \geq 0\} \cap [-2d, 2d],$$

which are compact subsets of $\mathbf{C}$. Notice (see [6]) that there exist constants $b_1 = a_1 = 1$ and $b_d = e^{G(2d)} > a_d = e^{G(0)} > 0$, $d \geq 2$ (NB: The function G considered in this paper differs from the G defined in [6] by a translation of z by $2d$), such that $S_\lambda = P_\lambda \cup C_\lambda$ looks like

$S_\lambda = C_\lambda =$ $-2d$ ———————— $2d$ and $P_\lambda = \emptyset$ if $\lambda \leq a_d$,

$S_\lambda =$ 

if $a_d < \lambda \leq b_d$,

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