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# DOUBLE HILBERT TRANSFORMS ALONG POLYNOMIAL SURFACES IN $\mathbb{R}^3$

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**1. Introduction.** In this paper we consider the $L^p$-boundedness of operators defined formally as

$$Hf(x,y,z)=\int_{|s|\le 1}\int_{|t|\le 1} f\big(x-s,y-t,z-P(s,t)\big)\frac{ds\,dt}{st},$$

where $P(s,t)$ is a polynomial in $s$ and $t$ with $P(0,0)=0$, and $\nabla P(0,0)=0$. We call $H$ the (local) double Hilbert transform along the surface $(s,t,P(s,t))$. The operator may be precisely defined for a Schwartz function $f$ by integrating where $\epsilon \le |s| \le 1$ and $\eta \le |t| \le 1$, and then taking the limit as $\epsilon,\eta \to 0$. The corresponding 1-parameter problem has been extensively studied (see [RS1], [RS2], and [S], for example). The type of operator that we are concerned with in this paper has been previously studied in [NW], [RS3], and [V]. In those works, operators that are in some ways more general than ours are considered, but only under an appropriate dilation invariance, which in our setting would force $P$ to be a monomial. If $P(s,t)=s^m t^n$, then according to [RS3] (see Section 5 below for the precise statement), for any $p$, $1<p<\infty$, $H$ is bounded in $L^p$ if and only if at least one of $m$ and $n$ is even.

Our present result is stated in terms of the Newton diagram of $P$, which we describe below. Recently Phong and Stein have shown how the Newton diagram also plays a decisive role in describing the mapping properties of certain degenerate Fourier integral operators (see [PS]).

Main theorem. *For any $p$, $1<p<\infty$,*

$$\|Hf\|_{L^p} \le A_p \|f\|_{L^p}$$

*if and only if for each $(m,n)$ that is a corner point of the Newton diagram corresponding to $P$, at least one of $m$ and $n$ is even.*

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