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EXISTENCE AND REGULARITY FOR HIGHER-DIMENSIONAL H-SYSTEMS

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1. Introduction. In this paper we are concerned with the existence and regularity of solutions of the degenerate nonlinear elliptic systems known as H-systems. For a given real-valued function H defined on (a subset of) $\mathbb{R}^{n+1}$, the associated H-system on a subdomain of $\mathbb{R}^n$ (we generally take the domain to be B, the unit ball) is given by

$$D_{x_i}\left(|Du|^{n-2}D_{x_i}u\right) = \sqrt{n^n}(H\circ u)u_{x_1}\times\cdots\times u_{x_n} \tag{1.1}$$

for a map u from B to $\mathbb{R}^{n+1}$. (Obviously for (1.1) to make sense classically, we look for $u\in C^2(B,\mathbb{R}^{n+1})$. As we discuss in Section 2, it also makes sense to look for a weak solution $u\in W^{1,n}(B,\mathbb{R}^{n+1})$ to (1.1) under suitable restrictions on H.) Here we use the summation convention, and the cross product $w_1\times\cdots\times w_n : \mathbb{R}^{n+1}\oplus\cdots\oplus\mathbb{R}^{n+1}\to\mathbb{R}^{n+1}$ is defined by the property that $w\cdot w_1\times\cdots\times w_n = \det W$ for all vectors $w\in\mathbb{R}^{n+1}$, where W is the $(n+1)\times(n+1)$ matrix whose first row is $(w^1,\dots,w^{n+1})$ and whose jth row is $(w^1_{j-1},\dots,w^{n+1}_{j-1})$ for $2\le j\le n+1$.

Equation (1.1) has a natural geometric property; namely, if u fulfills certain additional conditions, then it represents a hypersurface in $\mathbb{R}^{n+1}$ whose mean curvature at the point $u(x)$, for $x\in B$, is given by $H\circ u(x)$. Specifically, a map $u:B\to\mathbb{R}^{n+1}$ is called *conformal* if

$$u_{x_i}\cdot u_{x_j} = \lambda^2(x)\delta_{ij} \quad \text{on } B \tag{1.2}$$

for some real-valued function λ. If $u\in C^2(B,\mathbb{R}^3)$ is conformal, then it is possible to show that u defines a hypersurface in $\mathbb{R}^{n+1}$ which has mean curvature $H\circ u(x)$ at every *regular point* $u(x)$, meaning a point where $u_{x_1}\times\cdots\times u_{x_n}$ does not vanish. For $n=2$ this observation is the starting point for all existence results for parametric surfaces of prescribed mean curvature (cf. the references cited below for the Plateau problem). For $n\ge 3$ a derivation can be found in [DuF4, pp. 42 ff.].

We wish to discuss boundary value problems associated with (1.1), and we first consider the case $n=2$. Here the map u satisfies the *Plateau boundary condition* for a given rectifiable Jordan curve Γ in $\mathbb{R}^3$ if

$$u|_{\partial B} \text{ is a homeomorphism from } \partial B \text{ to } \Gamma. \tag{1.3}$$

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