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QUASI-ISOMETRIC RIGIDITY FOR $\mathrm{PSL}_2(\mathbb{Z}[1/p])$

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1. Introduction. Combining the work of many people yields a complete quasi-isometry classification of irreducible lattices in semisimple Lie groups (see [F] for an overview of these results). One of the first general results in this classification is the complete description, up to quasi-isometry, of all nonuniform lattices Λ in semisimple Lie groups of rank 1, proved by R. Schwartz [S1]. He shows that every quasi-isometry of such a lattice Λ is equivalent to a unique commensurator of Λ. (A *commensurator* of $\Lambda \subset G$ is an element $g \in G$ so that $g\Lambda g^{-1} \cap \Lambda$ has finite index in Λ.) We call this result *commensurator rigidity*, although it is a different notion than the commensurator rigidity of Margulis. In [FS] it was conjectured that commensurator rigidity, or at least a slightly weaker statement, "quasi-isometric if and only if commensurable," should apply to nonuniform lattices in a wide class of Lie groups. Here we prove that both of these statements are true for $\mathrm{PSL}_2(\mathbb{Z}[1/p])$.

In a different direction, B. Farb and L. Mosher proved analogous quasi-isometric rigidity results for the solvable Baumslag-Solitar groups. These groups are given by the presentation

$$\mathrm{BS}(1,n) = \langle a, b \mid aba^{-1} = b^n \rangle$$

and are not lattices in any Lie group.

The group $\mathrm{PSL}_2(\mathbb{Z}[1/p])$ is a nonuniform (i.e., noncocompact) lattice in the group $\mathrm{PSL}_2(\mathbb{R}) \times \mathrm{PSL}_2(\mathbb{Q}_p)$, analogous to the classical Hilbert modular group $\mathrm{PSL}_2(\mathbb{O}_d)$ in $\mathrm{PSL}_2(\mathbb{R}) \times \mathrm{PSL}_2(\mathbb{R})$. It is also a basic example of an S-arithmetic group. The proofs of Theorems A, B, and C (stated below) combine techniques from the two types of quasi-isometric rigidity results mentioned above. When we construct a space Ω_p on which $\mathrm{PSL}_2(\mathbb{Z}[1/p])$ acts properly, discontinuously, and cocompactly by isometries, we see that the horospheres forming the boundary components of Ω_p carry the geometry of the group $\mathrm{BS}(1, p)$. In this way the results of [FM] play a role in the quasi-isometric rigidity of $\mathrm{PSL}_2(\mathbb{Z}[1/p])$.

1.1. Statement of results. In this paper we prove the following quasi-isometric rigidity results for the finitely generated groups $\mathrm{PSL}_2(\mathbb{Z}[1/p])$, where p is a prime. Theorem A may be viewed as a strengthening of strong (Mostow) rigidity for $\mathrm{PSL}_2(\mathbb{Z}[1/p])$. [M]

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