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TORUS FIBRATIONS OF CALABI-YAU HYPERSURFACES IN TORIC VARIETIES

ILIA ZHARKOV

1. Introduction. Strominger, Yau, and Zaslow [SYZ] conjectured that any Calabi-Yau manifold X having a mirror partner $X^\vee$ should admit a special Lagrangian fibration $\pi : X \to B$. (A mathematical account of their construction can be found in [M].) If so, the mirror manifold $X^\vee$ is obtained by finding some suitable compactification of the moduli space of flat $U(1)$-bundles along the nonsingular fibers, which restricts the fibers to be tori. More precisely, if $B_0 \subseteq B$ is the largest set such that $\pi_0 = \pi|_{\pi^{-1}(B_0)}$ is smooth, then $X^\vee$ should be a compactification of the dual fibration $R^1\pi_{0*}(\mathbb{R}/\mathbb{Z}) \to B_0$.

The conjecture is trivial in the elliptic curve case. On a $K3$-surface, the hyper-Kähler structure translates the theory of special Lagrangian T^2-fibrations to the standard theory of elliptic fibrations in another complex structure. However, little progress has been made thus far in higher dimensions. Gross and Wilson [GrW] have worked out some aspects of the conjecture for the Voisin-Borcea 3-folds of the form $(K3 \times T^2)/\mathbb{Z}_2$. Also Bryant [Br] found special Lagrangian tori inside some hypersurfaces in $\mathbb{CP}^n$ for $n = 2, 3$, and 4 as components of the set of real points, though it is unclear if there is a fibration. But the general question of finding special Lagrangian fibrations on Calabi-Yau manifolds still remains open.

We examine the case of regular anticanonical hypersurfaces in smooth toric varieties. The main result of this paper is that such a hypersurface in a neighborhood of the large complex structure admits a torus fibration over a sphere. Unfortunately, we were unable to control the fibers to be special Lagrangians. However we argue that on some open patches the fibers do possess some calibration property.

The application of the moment map for constructing torus fibrations was observed by Gross and Wilson [GrW] and also by Morrison and collaborators using Batyrev's construction (see [B]) for mirror symmetry. Batyrev showed that toric varieties X_Δ with ample anticanonical bundles are given by reflexive polyhedra. Such a polyhedron Δ contains a unique integral interior point $\{0\}$. A Calabi-Yau hypersurface $Y \subset X_\Delta$ is defined by an equation in the form $\sum_{\omega\in\Delta(\mathbb{Z})} a_\omega x^\omega = 0$, where ω runs over the integral points in Δ. The image of Y under the moment map $\mu : X_\Delta \to \Delta$ has the shape of an amoeba (cf. [GKZ, Chap. 6]), a blob with holes around some lattice points ω in $\Delta(\mathbb{Z})$. The sizes of the holes are determined by the corresponding coefficients a_ω. If Y is near the large complex structure, then $a_{\{0\}}$ is large and $\mu(Y)$ has exactly

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