Vol. 101, No. 2 DUKE MATHEMATICAL JOURNAL © 2000

ON THE MORGAN-SHALEN COMPACTIFICATION OF THE SL$(2,\mathbb{C})$ CHARACTER VARIETIES OF SURFACE GROUPS

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1. Introduction. Let Σ be a closed, compact, oriented surface of genus $g \geq 2$ and fundamental group Γ. Let $\mathscr{X}(\Gamma)$ denote the SL$(2,\mathbb{C})$ character variety of Γ, and $\mathscr{D}(\Gamma) \subset \mathscr{X}(\Gamma)$ the closed subset consisting of conjugacy classes of discrete, faithful representations. Then $\mathscr{X}(\Gamma)$ is an affine algebraic variety admitting a compactification $\overline{\mathscr{X}(\Gamma)}$ (due to Morgan and Shalen [MS1]), whose boundary points $\partial\mathscr{X}(\Gamma) = \overline{\mathscr{X}(\Gamma)} \setminus \mathscr{X}(\Gamma)$ correspond to elements of $\mathscr{PL}(\Gamma)$, the space of projective classes of length functions on Γ with the weak topology.

Choose a metric σ on Σ, and let $\mathscr{M}_{\text{Higgs}}(\sigma)$ denote the moduli space of semistable rank-2 Higgs pairs on Σ (σ) with trivial determinant, as constructed by Hitchin [H]. Then $\mathscr{M}_{\text{Higgs}}(\sigma)$ is an algebraic variety, depending on the complex structure defined by σ (cf. [Si]). By the theorem of Donaldson [D], $\mathscr{M}_{\text{Higgs}}(\sigma)$ is homeomorphic to $\mathscr{X}(\Gamma)$, though not complex-analytically so. Let us denote this map $h : \mathscr{X}(\Gamma) \to \mathscr{M}_{\text{Higgs}}$ (we henceforth assume the choice of base point σ).

We define a compactification of $\mathscr{M}_{\text{Higgs}}$ as follows: Let QD (more precisely, $QD(\sigma)$) denote the finite-dimensional complex vector space of holomorphic quadratic differentials on Σ. Then there is a surjective, holomorphic map $\mathscr{M}_{\text{Higgs}} \to QD$ taking the Higgs field Φ to $\varphi = \det\Phi$. We compose this with the map

$$\varphi \longrightarrow \frac{4\varphi}{1+4\|\varphi\|},$$

where $\|\varphi\| = \int_\Sigma |\varphi|$, and obtain

$$\widetilde{\det} : \mathscr{M}_{\text{Higgs}} \longrightarrow BQD = \{\varphi \in QD : \|\varphi\| < 1\}.$$

Let $SQD = \{\varphi \in QD : \|\varphi\| = 1\}$ be the space of normalized holomorphic quadratic differentials. We then define $\overline{\mathscr{M}_{\text{Higgs}}} = \mathscr{M}_{\text{Higgs}} \cup SQD$ with the topology given via the

Received 6 August 1998.

1991 *Mathematics Subject Classification*. Primary 58E20; Secondary 20E08, 30F30, 32G13.

Daskalopoulos's work partially supported by National Science Foundation grant number DMS-9803606.

Dostoglou's work partially supported by the Research Board of the University of Missouri and the Arts and Science Travel Fund of the University of Missouri, Columbia.

Wentworth's work partially supported by National Science Foundation grant number DMS-9971860 and a Sloan Fellowship.