Vol. 101, No. 1 DUKE MATHEMATICAL JOURNAL © 2000

TANGENT VECTORS TO HECKE CURVES ON THE MODULI SPACE OF RANK 2 BUNDLES OVER AN ALGEBRAIC CURVE

JUN-MUK HWANG

The moduli space of semistable bundles of a fixed determinant over an algebraic curve has been studied by many authors from various points of view. Especially well-understood is the case of rank 2 bundles. In this case, quite a detailed study of the geometry of the moduli space has been done in [Be], [BV], and [DR].

This moduli space is a Fano variety of Picard number 1. In a series of joint works with N. Mok, we have studied the geometry of Fano manifolds of Picard number 1 by investigating the projective geometry of the variety of tangent directions of the minimal rational curves (see [HM3] for a survey). Our aim here is to apply this study to the moduli space M_i of rank 2 bundles of a fixed determinant of degree $i = 0, 1$ over an algebraic curve of genus $g \geq 2$. In this case, the minimal rational curves in the sense of [HM3] turn out to be "Hecke curves," originally introduced by Narasimhan and Ramanan [NR1], [NR2]. Using the results in [NR2, Section 5], we study the variety of tangent vectors to Hecke curves through a fixed point on M_i. As a consequence of this study and our previous work [H], we get the following result, which seems to have been speculated by experts in this field.

THEOREM 1. *Let M_1 be the moduli space of stable bundles of rank 2 with a fixed determinant of odd degree over an algebraic curve of genus $g \geq 2$. Then the tangent bundle of M_1 is stable.*

Our next result is on the deformation rigidity of generically finite morphisms over M_i. A result of this type is expected for many Fano manifolds of Picard number 1, and the result here can be regarded as an example (see [HM1] for a general discussion). To streamline the presentation, we assume $g \geq 4$ for M_0 and $g \geq 5$ for M_1 in the next theorem. It may be possible to extend our result to some cases of lower genus, but one would need a different idea to cover all the cases, especially the case of $g = 2$.

THEOREM 2. *Assume $g \geq 4$ for $i = 0$ and $g \geq 5$ for $i = 1$. Let Y be any projective manifold of dimension $3g-3$, and let $f : Y \to M_i$ be a surjective holomorphic map.*

Received 20 April 1998. Revision received 2 March 1999.

1991 *Mathematics Subject Classification*. Primary 14H60; Secondary 14J45.

Author supported by Seoul National University Research Fund and by grant number 98-0701-01-5-L from the Korea Science and Engineering Foundation.